République Algérienne Démocratique et Populaire
Ministère de l'Enseignement Supérieur et de la Recherche Scientifique
Université Yahia Fares de Médéa
Faculté des Sciences
Département de Mathématiques et Informatique

Mémoire de Master

présenté en vue de l'obtention du

DIPLÔME DE MASTER EN MATHÉMATIQUES

Spécialité : Analyse Combinatoire, Probabilité et Applications

Par

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Étude stochastique de la production du venin de scorpion à l'aide des processus de naissance et de mort

Soutenue publiquement le 23/06/2026, devant le jury composé de :

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Année Universitaire 2025/2026

Democratic and Popular Republic of Algeria
Ministry of Higher Education and Scientific Research
Yahia Fares University of Médéa
Faculty of Sciences
Department of Mathematics and Computer Science

Master's Dissertation

submitted in partial fulfillment of the requirements for the

MASTER'S DEGREE IN MATHEMATICS

Specialty : Combinatorial Analysis, Probability and Applications

By

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Stochastic Study of Scorpion Venom Production Using Birth-Death Processes

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Academic Year 2025/2026

Acknowledgments

This humble work extends its sincerest gratitude, immense appreciation, and utmost respect to the esteemed supervisor of this dissertation, Dr. Kamel Toumache, who was for me the best guide and supporter throughout the duration of this research ; he devoted much of his time and effort, and his insightful guidance, valuable advice, and continuous support were the compass that guided my steps at every stage of this work, so I pray that Allah blesses his knowledge and time, rewards him abundantly, and makes him a beacon illuminating the path for his students after him.

In this context, I cannot but extend my profound gratitude and deep respect to the esteemed members of the examination committee who honored me by accepting to evaluate this humble work, namely : Dr. TABLENNEHAS Kamel, who graciously accepted to chair the committee, and Dr. BENTERKI Abdessalem, who kindly participated as an examiner, for their valuable presence alongside this work and their review of it with their vast knowledge and insightful academic perspectives is a great honor for me, and I ask Allah to reward each of them abundantly for the time and effort they devoted to enriching and evaluating this research.

I would particularly like to thank Professor Nabil Ould Mohyeddine and Professor Chadhli Oussama for their invaluable support, guidance, and the knowledge they so generously shared, which greatly contributed to shaping my academic path. Finally, I also extend my

sincere thanks and deep gratitude to all my esteemed professors, from whom I learned throughout my academic journey, who were for me illuminating beacons on the path of knowledge and planted in me the love of seeking further knowledge and the spirit of research and inquiry, so may Allah bless your paths and increase you in knowledge and stature. To everyone who supported and helped me, near or far, I ask Allah to make this work purely for His sake and beneficial to knowledge and society ; He is indeed the Guardian and the One capable of that, and all praise is due to Allah, Lord of all worlds.

Dedication

I dedicate the fruit of this humble effort, which would not have come to light without the grace of Allah and then the support of those around me, to those who have been and continue to be my support and strength in this life, and to those who illuminated my path with knowledge and wisdom. Firstly, to my beloved parents : to my dear father, Abdelatif Djelloul, who has been and remains my highest example of patience and giving ; to my beloved father, Mouhamed Abdellatif, who instilled in me the love of knowledge and ambition from my earliest days ; and to my tender mother, Safia Drif, who has been and remains a source of tenderness, prayer, and unwavering support—to you I owe all my love and loyalty, and I pray that Allah is pleased with you, blesses your lives, and grants you health and well-being. To my dear brothers, Abdel Halim, Farouk, and Fares, who have been my strong support and constant encouragement, companions on the path through moments of hardship and joy—you are an invaluable treasure. To my dear friends, who are like sisters to me, you were the best companions and friends at every step, so to you goes all my love and appreciation. To my closest family members, who have been and continue to be the best support and help, and to everyone who was a support and strength to me during this long journey, whether through moral support, advice, or encouragement—you have all made this achievement possible.

I ask Allah Almighty to bless you all, to make this humble work purely for His sake, beneficial to knowledge and society, and to grant me success in what He loves and pleases ; He is indeed the Guardian and the One capable of that, and all praise is due to Allah, Lord of all worlds.

ABSTRACT

This thesis presents a novel, rigorous stochastic mathematical model for the optimization of scorpion venom production, explicitly integrating the discrete and random nature of molecular synthesis and degradation with the biological realities of the venom gland. The model is formulated as a continuous-time Markov chain (CTMC) using a linear birth-death process on the countable state space $\mathbb{N}_0$, where the state variable $X(t)$ represents the number of venom units at time t. The dynamics are governed by three independent Poisson processes : venom synthesis (birth) with state-dependent rates influenced by temperature T and nutrition level N_u, spontaneous degradation (death) with temperature-dependent rates following an Arrhenius-type function, and periodic milking (catastrophe) which removes a fixed batch of m units at a constant rate γ, provided sufficient venom is available. The infinitesimal generator $\mathbf{Q}$ is explicitly constructed, and the forward Kolmogorov equations (master equation) are derived. Through rigorous analytical derivation using probability generating functions, we establish that a unique stationary distribution exists if and only if the traffic intensity $\rho = \lambda/\mu < 1$; for the general case with milking, we obtain the explicit integral representation

$$G(z) = \exp\left(\frac{\gamma}{\mu}\int_1^z \frac{1-w^{-m}}{\lambda(w-1)-\mu}dw\right).$$

Exact differential equations for the mean and variance are derived, and extinction probabilities as well as expected extinction times are fully characterized for both the pure birth-death case and the case with milking. To overcome analytical intractability, the Gillespie direct method is implemented in Python to generate exact sample paths of the stochastic process, validated against experimental data from Yaqoob et al. (2016) yielding excellent agreement ($R^2 = 0.94$). Extensive simulations with 10^4 independent realizations quantify the effects of environmental factors, revealing a unimodal temperature optimum at $\approx 32°$C where venom yield increases by 31% compared to baseline, and a concave nutrition dependence where high nutrition enhances mean venom quantity by 35% at 30 days. Global sensitivity analysis using Latin Hypercube sampling identifies the birth rate λ (PRCC = 0.72) and death rate μ (PRCC = -0.68) as the most influential parameters.

The optimization problem is formulated to maximize the expected net profit

$$\mathcal{J}(T, N_u) = R_{\text{venom}}\mathbb{E}[Y_{\text{total}}] - (C_{\text{temp}}(T) + C_{\text{feed}}(N_u))\,\mathbb{E}[T_{\text{ext}}],$$

balancing revenue against temperature control and feeding costs. Using a two-stage numerical approach combining grid search with the Nelder-Mead simplex algorithm, the optimal parameters are identified as $T^* = 32.6°\text{C}$ and $N_u^* = 0.88$ (i.e., 12% below ad libitum), yielding a 12% improvement in daily net profit over the baseline strategy (62.3 vs. 58.7 monetary units/day). Economic analysis demonstrates a Return on Investment (ROI) of approximately 20.8% and a payback period of 4.8 years for implementing the optimal controls. The principal contributions of this thesis are threefold : first, the analytical solution for the stationary distribution of a birth-death process with catastrophes of size $m \geq 2$ explicitly incorporating temperature and nutrition dependencies ; second, the integration of exact stochastic simulation, moment closure approximations, and numerical optimization within a unified framework ; and third, the provision of evidence-based operational recommendations including precise temperature control ($32.5 \pm 1.0°\text{C}$), reduced nutrition to 88% of maximum, an 11-day milking interval, a batch size of 6 units, and an initial colony size of at least 10 individuals. This work successfully demonstrates that moderate reductions in controllable inputs can yield substantial improvements in net profit, moving venom production from purely empirical practice to a quantitatively optimized, decision-support framework grounded in stochastic chemical kinetics and agricultural biotechnology. The findings open new avenues for the application of stochastic processes to venom research, and the Python code developed is made available to facilitate further experimental validation and adaptive management strategies.

RÉSUMÉ

Cette thèse présente un nouveau modèle mathématique stochastique rigoureux pour l'optimisation de la production de venin de scorpion, intégrant explicitement la nature discrète et aléatoire de la synthèse et de la dégradation moléculaires avec les réalités biologiques de la glande à venin. Le modèle est formulé comme une chaîne de Markov en temps continu (CTMC) utilisant un processus de naissance et de mort linéaire sur l'espace d'états dénombrable $\mathbb{N}_0$, où la variable d'état $X(t)$ représente le nombre d'unités de venin au temps t. La dynamique est régie par trois processus de Poisson indépendants : la synthèse du venin (naissance) avec des taux dépendant de l'état et influencés par la température T et le niveau nutritionnel N_u, la dégradation spontanée (mort) avec des taux dépendant de la température suivant une loi de type Arrhenius, et la traite périodique (catastrophe) qui élimine un lot fixe de m unités à un taux constant γ, à condition que suffisamment de venin soit disponible. Le générateur infinitésimal $\mathbf{Q}$ est explicitement construit, et les équations de Kolmogorov prospectives (équation maîtresse) sont dérivées. Grâce à une dérivation analytique rigoureuse utilisant les fonctions génératrices des probabilités, nous établissons qu'une distribution stationnaire unique existe si et seulement si l'intensité du trafic $\rho = \lambda/\mu < 1$; pour le cas général avec traite, nous obtenons la représentation intégrale explicite

$$G(z) = \exp\left(\frac{\gamma}{\mu}\int_1^z \frac{1-w^{-m}}{\lambda(w-1)-\mu}dw\right).$$

Les équations différentielles exactes pour la moyenne et la variance sont dérivées, et les probabilités d'extinction ainsi que les temps d'extinction attendus sont entièrement caractérisés pour le cas de la naissance-mort pure et pour le cas avec traite. Pour surmonter l'intraitabilité analytique, la méthode directe de Gillespie est implémentée en Python pour générer des trajectoires exactes du processus stochastique, validée par des données expérimentales de Yaqoob et al. (2016) avec une excellente concordance ($R^2 = 0.94$). Des simulations extensives avec 10^4 réalisations indépendantes quantifient les effets des facteurs environnementaux, révélant un optimum de température unimodal à ≈ 32°C où le rendement en venin augmente de 31% par rapport à la référence, et une dépendance nutritionnelle concave où une nutrition élevée améliore la quantité moyenne de venin de 35% à 30 jours. L'analyse de sensibilité globale utilisant l'échantillonnage hypercube latin identifie le taux de naissance λ (PRCC = 0.72) et le taux de mort μ (PRCC = -0.68) comme les paramètres les plus influents. Le problème d'optimisation est formulé pour

maximiser le bénéfice net attendu

$$\mathcal{J}(T, N_u) = R_{\text{venin}}\mathbb{E}[Y_{\text{total}}] - (C_{\text{temp}}(T) + C_{\text{alim}}(N_u))\,\mathbb{E}[T_{\text{ext}}],$$

équilibrant les revenus avec les coûts de contrôle de la température et d'alimentation. En utilisant une approche numérique en deux étapes combinant une recherche sur grille avec l'algorithme du simplexe de Nelder-Mead, les paramètres optimaux sont identifiés comme $T^* = 32.6°\text{C}$ et $N_u^* = 0.88$ (c'est-à-dire 12% en dessous de l'ad libitum), produisant une amélioration de 12% du bénéfice net quotidien par rapport à la stratégie de référence (62.3 contre 58.7 unités monétaires/jour). L'analyse économique démontre un retour sur investissement (ROI) d'environ 20.8% et une période de récupération de 4.8 ans pour la mise en œuvre des contrôles optimaux. Les principales contributions de cette thèse sont triples : premièrement, la solution analytique de la distribution stationnaire d'un processus de naissance-mort avec catastrophes de taille $m \geq 2$ incorporant explicitement les dépendances à la température et à la nutrition ; deuxièmement, l'intégration de la simulation stochastique exacte, des approximations de fermeture des moments et de l'optimisation numérique dans un cadre unifié ; et troisièmement, la fourniture de recommandations opérationnelles basées sur des preuves incluant un contrôle précis de la température ($32.5 \pm 1.0°\text{C}$), une nutrition réduite à 88% du maximum, un intervalle de traite de 11 jours, une taille de lot de 6 unités et une taille de colonie initiale d'au moins 10 individus. Ce travail démontre avec succès que des réductions modérées des intrants contrôlables peuvent générer des améliorations substantielles du bénéfice net, faisant passer la production de venin d'une pratique purement empirique à un cadre d'aide à la décision quantitativement optimisé, ancré dans la cinétique chimique stochastique et la biotechnologie agricole. Ces résultats ouvrent de nouvelles voies pour l'application des processus stochastiques à la recherche sur le venin, et le code Python développé est mis à disposition pour faciliter la validation expérimentale et les stratégies de gestion adaptative.

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List of Notations

Symbol	Meaning	Unit	Section
i or x	Current venom quantity	dimensionless	2.1
t	Time	seconds, hours, days	2.1
$X(t)$	Number of venom units at time t	dimensionless	2.1
S	Countable state space, $S = \mathbb{N}_0$	—	1.3.1
x_0	Initial venom quantity, $X(0) = x_0$	dimensionless	2.8
T	Temperature	°C	2.1
N_u	Nutrition level	dimensionless	2.1
T_{opt}	Optimal temperature for venom production	°C	2.1
$T_{\min}, T_{\max}$	Minimum / maximum viable temperature	°C	2.4
T_{ref}	Reference temperature for $h(T)$	°C	2.5
T_{amb}	Ambient temperature	°C	4.1
σ_T	Width of the Gaussian temperature response	°C	2.4
$f_{\max}$	Maximum per-capita synthesis rate	time^{-1}	2.4
K_N	Half-saturation constant (nutrition)	dimensionless	2.4
h_0	Baseline degradation rate at T_{ref}	time^{-1}	2.5
Q_{10}	Temperature coefficient for degradation	dimensionless	2.5
$f(T)$	Temperature dependence function (birth rate)	dimensionless	2.1
$g(N_u)$	Nutrition dependence function (birth rate)	dimensionless	2.1
$h(T)$	Temperature dependence of degradation rate	time^{-1}	2.1
$\lambda_i(T, N_u)$	Birth rate (venom production rate) at state i	time^{-1}	2.1
$\mu_i(T)$	Death rate (venom degradation rate) at state i	time^{-1}	2.1
γ	Milking rate (external removal rate)	time^{-1}	1.3.2
γ_i	State-dependent milking rate, $\gamma \mathbf{1}_{\{i \geq m\}}$	time^{-1}	2.6
m	Number of venom units removed per milking event	dimensionless	1.3.2
$p_{ij}(t)$	Transition probability from i to j in time t	dimensionless	1.1

Symbol	Meaning	Unit	Section
$\mathbf{P}(t)$	Transition (semigroup) matrix	dimensionless	1.1
δ_{ij}	Kronecker delta	dimensionless	1.2
Q	Generator matrix (Q-matrix)	time^{-1}	1.3.1
q_{ij}	Generator entry, transition rate from i to j	time^{-1}	1.2
q_i	Total exit rate from state i, $q_i = -q_{ii}$	time^{-1}	1.2
$P_i(t)$	Probability of being in state i at time t	dimensionless	1.3.2
π_i	Stationary probability of state i	dimensionless	2.3.1
ρ	Traffic intensity, $\rho = \lambda/\mu = f(T)g(N_u)/h(T)$	dimensionless	2.3.1
$G(z,t)$	Probability generating function (PGF)	dimensionless	2.9
$G(z)$	Stationary PGF	dimensionless	2.14
$\mathbb{E}[X(t)]$	Expected venom quantity at time t	dimensionless	2.3.2
$\mathrm{Var}[X(t)]$	Variance of venom quantity at time t	dimensionless^2	2.3.2
$M_k(t)$	k-th moment of $X(t)$	dimensionless	2.16
$\sigma^2(t)$	Variance, $\sigma^2 = M_2 - M_1^2$	dimensionless^2	2.16
$\pi_{\geq m}$	$\mathbb{P}(X \geq m)$	dimensionless	2.6
$M_1^{(m)}$	$\mathbb{E}[X\mathbf{1}_{\{X \geq m\}}]$	dimensionless	2.6
κ_r	r-th cumulant of $X(t)$	varies	1.4.10
$K(\theta, t)$	Cumulant generating function	dimensionless	1.4.10
Φ, ϕ	Standard normal CDF / PDF	dimensionless	2.3.3
q_i	Extinction probability starting from state i	dimensionless	2.17
T_{ext}	Extinction time, $T_{\text{ext}} = \inf\{t \geq 0 : X(t) = 0\}$	days	2.3.4
m_i	Expected time to extinction from state i	days	2.18
T_{ij}	First passage time from state i to j	time	2.19
$a_j(\mathbf{x})$	Propensity function for reaction j	time^{-1}	1.4.3
$a_0(\mathbf{x})$	Total propensity function	time^{-1}	1.4.3
$\boldsymbol{\nu}_j$	State-change vector for reaction j	dimensionless	1.4.3
τ	Time to next reaction	time	1.4.3
r_1, r_2	Uniform random numbers on $[0, 1]$	dimensionless	1.4.3
$\mathcal{P}(m)$	Poisson random variable with mean m	dimensionless	1.4.3
$\mathcal{N}(\mu, \sigma^2)$	Normal random variable with mean μ, variance σ^2	varies	1.4.3
Y_{total}	Total venom yield over colony lifetime	venom units	4.1
$Y_{\text{ss}}(T, N_u)$	Expected total yield under stationary distribution	venom units	4.1
$\mathbb{E}[X]_\pi$	Stationary mean of X	dimensionless	4.1
$\tau_{\text{ext}}(T, N_u)$	Expected extinction time (closure approximation)	days	4.1

Symbol	Meaning	Unit	Section
$\mathcal{J}(T, N_u)$	Expected net profit function	monetary units/day	4.1
$\widetilde{\mathcal{J}}$	Net revenue, $\widetilde{\mathcal{J}} = \mathcal{J} + C_{\text{milk}}$	monetary units/day	4.1
R_{venom}	Revenue per venom unit	monetary units	4.1
$C_{\text{temp}}(T)$	Temperature control cost	monetary units/day	4.1
$C_{\text{feed}}(N_u)$	Feeding cost	monetary units/day	4.1
$C_{\text{milk}}(\gamma)$	Milking cost	monetary units/day	4.1
c_0, c_1	Fixed / variable temperature control cost coefficients	varies	4.1
c_f	Feeding cost coefficient	monetary units/day	4.1
c_m	Milking cost coefficient	monetary units/day^2	4.1
T^*, N_u^*	Optimal temperature and nutrition level	°C, dimensionless	4.2.2
γ^*, m^*	Optimal milking rate and batch size	time^{-1}, dimensionless	4.2.2
$\alpha_{\text{NM}}, \chi_{\text{NM}}, \beta_{\text{NM}}, \sigma_{\text{NM}}$	Nelder–Mead reflection, expansion, contraction, shrinkage coefficients	dimensionless	4.2.1
ROI	Return on investment	dimensionless (percentage)	
P	Payback period, $P = I_0/\Delta\Pi$	years	4.3.2
I_0	Initial investment	monetary units	4.3.1
Π_{annual}	Annual net profit	monetary units/year	4.3.1
$\Delta\Pi$	Incremental annual profit	monetary units/year	4.3.1

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GENERAL INTRODUCTION

Scorpion venom represents one of the most sophisticated and pharmacologically valuable secretions in the natural world. It is a complex cocktail of peptides, proteins, salts, and small molecules, evolved over millions of years for prey immobilization and defense. In recent decades, this venom has emerged as a rich source of lead compounds for drug discovery. The most iconic example is **Chlorotoxin**, isolated from the venom of *Leiurus quinquestriatus*, which exhibits specific binding to glioma cells with an exceptionally high affinity, characterized by a dissociation constant $K_d = 2.4 \pm 0.3\,\text{nM}$ [8]. This property has made chlorotoxin a valuable tool for imaging and targeted therapy in neuro-oncology. Beyond chlorotoxin, scorpion venom contains a plethora of neurotoxins, antimicrobial peptides, and enzyme inhibitors with potential applications in cardiology, immunology, and infectious disease treatment. Consequently, the global market for venom-derived pharmaceuticals is projected to experience significant growth in the coming decade, driving the need for efficient, sustainable, and scalable production methods. Despite this immense therapeutic potential, the commercial production of scorpion venom has remained largely artisanal and empirical, guided by trial and error rather than by quantitative, predictive science. This thesis addresses this critical gap by developing a rigorous stochastic mathematical model for venom production, grounded in the biological realities of the scorpion venom gland. The scorpion venom apparatus is located in the **telson**, the terminal segment of the metasoma (the tail). It consists of a pair of elongated, sac-like venom glands enveloped by striated muscle fibers. Contraction of these muscles forces stored venom through a cuticle-lined duct to the aculeus (stinger) [10, 11]. The functional unit of the gland is its secretory epithelium, and ultrastructural studies have definitively characterized the mode of secretion as **apocrine** [10, 12, 14]. Unlike merocrine secretion (exocytosis without cell loss) or holocrine secretion (cell disintegration), apocrine secretion involves the budding and pinching-off of the apical portion of the secretory cell's cytoplasm. This process releases membrane-bound vesicles into the gland lumen. Critically, these vesicles contain not only the concentrated venom proteins and peptides but also a payload of cellular components, including portions of the cytosol, organelles, and—most significantly for modern research—**messenger RNA (mRNA) transcripts** [12, 7, 14]. This apocrine mechanism has a transformative implication: as shown by Vonk et al. (2021) [7], RNA extracted directly from venom (e.g., from *Het-*

erometrus laoticus) yields transcriptomes that are remarkably similar ($R^2 > 0.95$) to those obtained from dissected telson tissue, enabling non-lethal, longitudinal studies of venom gene expression. The quantity and composition of scorpion venom are not static; they exhibit significant plasticity in response to environmental and intrinsic factors [5]. Understanding these modulators is essential for any optimization effort. Temperature, as ectotherms, scorpions are profoundly influenced by ambient temperature. Metabolic rate, activity levels, and venom regeneration are temperature-dependent [15]. Field and laboratory observations consistently show that venom yield peaks during warmer months (30–40 °C) and declines significantly at lower temperatures (15–20 °C). Pearson correlation analysis reveals a significant positive correlation between venom production and temperature, with $r^2 = 0.957$ for *Androctonus finitimus* and $r^2 = 0.955$ for *Hottentotta tamulus* [6]. The relationship follows a characteristic unimodal (bell-shaped) curve, with an optimal temperature T_{opt} where production is maximized. Diet and nutrition, venom production is a metabolically costly process [5]. A controlled feeding study by Yaqoob et al. (2016) [6] demonstrated a clear hierarchy in venom yield based on prey type. Scorpions fed grasshopper nymphs produced the highest yield ($8.8 \pm 0.36\,\mu$l for *A. finitimus*), followed by grasshopper adults, house crickets, moths, and finally house flies ($5.5 \pm 0.28\,\mu$l). This indicates that prey with higher nutritional value (protein/lipid content) provides the necessary biosynthetic precursors for optimal venom gland function. Milking frequency and technique, repeated venom extraction (milking) leads to progressive gland depletion. A study tracking consecutive extractions at 15-day intervals showed a decline in yield from $9\,\mu$l to $4.5\,\mu$l over eight extractions for *A. finitimus* [6]. The method of extraction is equally critical. **Electrical stimulation** (typically square wave, 18 V, 10% duty factor, 45 Hz) yields significantly higher volumes ($7.6 \pm 0.26\,\mu$l for *A. finitimus*) and greater molecular diversity (12 peptide peaks by MALDI-TOF) compared to manual extraction ($2.1 \pm 0.26\,\mu$l and 5 peaks). Electrically extracted venom is considered "true venom," dense and opaque white, while manually extracted venom is often a watery "prevenom" deficient in many high-molecular-weight neurotoxins [6, 7]. The central problem addressed by this thesis is the absence of a unified, quantitative, optimization-based framework for scorpion venom production that integrates the stochastic dynamics of molecular synthesis and degradation, the influence of environmental factors (temperature, nutrition), and the discrete nature of milking events. A critical review of existing mathematical models for biological production systems (Section 1.2) reveals a significant gap: deterministic models (exponential growth $dx/dt = rx$, logistic growth $dx/dt = rx(1 - x/K)$) are analytically simple but fail to capture the discrete nature of molecule counts and the intrinsic randomness (stochasticity) of molecular interactions, as noted by Gillespie (2007) [3] and Wilkinson (2006) [4], the deterministic solution depends only on the difference $\lambda - \mu$ and not on the individual values, and it cannot account for extinction events or variability between realizations; age-structured models (McKendrick–Von Foerster equation) provide more detail but require extensive age-specific data and remain deterministic in nature;

stochastic birth–death processes have been applied to other biological systems, but no existing model incorporates temperature-dependent birth rates $f(T)$, nutrition-dependent saturation $g(N_u)$, and discrete, state-dependent milking events (removal of m units at rate γ) within a single, analytically tractable framework, furthermore, the apocrine secretion mechanism and its implications for stochasticity have not been previously modeled. Thus, this thesis is the first to develop a comprehensive stochastic model that explicitly links environmental control variables to the probabilistic dynamics of venom production, providing a foundation for rational optimization. To address the identified gap, this thesis formulates and answers the following primary research questions: first, formulation, how can the discrete, random processes of venom synthesis (birth), spontaneous degradation (death), and periodic milking (catastrophe) be mathematically formalized as a continuous-time Markov chain (CTMC) with state-dependent rates that depend on temperature T and nutrition N_u; second, analytical properties, what are the key analytical properties of this stochastic process, under what condition (e.g., $\rho = \lambda/\mu < 1$) does a unique stationary distribution exist, what is the probability and expected time to extinction for a venom-producing colony; third, computational implementation, how can the Gillespie algorithm (exact stochastic simulation) be implemented to generate sample paths of the process and to validate analytical approximations such as moment closure; fourth, optimization, given realistic cost functions for temperature control, feeding, and milking, what is the optimal temperature T^* and nutrition level N_u^* that maximize the expected net profit over the colony's lifetime, what is the optimal milking schedule (rate γ and batch size m); fifth, economic viability, what is the Return on Investment (ROI) and payback period for implementing the optimal control strategy in a commercial venom farm. The primary objective is to develop a decision-support tool that moves venom production from empirical practice to evidence-based, quantitative management. To achieve these objectives, the thesis employs a multi-scale methodological approach: stochastic modeling using CTMC and birth–death processes (Chapter 2); analytical derivations of stationary distributions via probability generating functions and detailed balance/global balance equations; exact stochastic simulation using the Gillespie direct method (Chapter 3); moment closure approximations (Gaussian/normal closure) to obtain tractable ODEs for mean and variance; numerical optimization using grid search combined with the Nelder–Mead simplex algorithm; and economic analysis using standard capital budgeting techniques (ROI, payback period). The principal innovations of this work are threefold: first, an analytical solution for the stationary distribution of venom production that explicitly includes milking events (catastrophes) of size $m \geq 2$, derived from the global balance equations and expressed in terms of an integral of the probability generating function (Theorem 2.3); second, an optimization algorithm that determines the optimal milking schedule (frequency γ and batch size m) and environmental conditions (T, N_u) to maximize total lifetime yield and economic profit, validated against experimental data from Yaqoob et al. (2016) [6] and Vonk et al. (2021) [7]; and third,

a comprehensive economic analysis showing that the optimized strategy ($T^* = 32.6°\text{C}$, $N_u^* = 0.88$, $\gamma = 0.09\,\text{day}^{-1}$, $m = 6$) yields an ROI of approximately 20.8% and a payback period of 4.8 years, representing a 12% improvement in net profit over baseline. This thesis is organized into five coherent chapters: Chapter 1 provides a comprehensive review of the biological foundations of scorpion venom production (gland anatomy, apocrine secretion, environmental modulators, milking techniques) and the mathematical foundations of CTMC, Kolmogorov equations, the Gillespie algorithm, and moment closure techniques, identifying the critical gap in existing models; Chapter 2 formulates the original stochastic birth–death model with state-dependent rates $\lambda_i = f(T)g(N_u)i$, $\mu_i = h(T)i$, and milking rate $\gamma_i = \gamma \mathbf{1}_{\{i \geq m\}}$, and derives analytical properties including stationary distribution (using PGF), extinction probability, mean and variance dynamics, and moment closure approximations; Chapter 3 presents the Python implementation of the Gillespie algorithm, defines simulation scenarios (baseline, temperature variation, nutrition variation, milking frequency), and reports extensive numerical results including typical trajectories, stationary distributions, temperature and nutrition dependence, extinction characteristics, global sensitivity analysis (Latin Hypercube sampling, PRCC), and validation against experimental data; Chapter 4 formulates the optimization problem to maximize expected net profit $\mathcal{J}(T, N_u) = R_{\text{venom}}\mathbb{E}[Y_{\text{total}}] - (C_{\text{temp}}(T) + C_{\text{feed}}(N_u))\mathbb{E}[T_{\text{ext}}]$, presents one-dimensional (temperature) and two-dimensional (temperature and nutrition) optimization results using Nelder–Mead, and provides economic analysis (ROI, payback period) and evidence-based operational recommendations for venom farms; and finally, the General Conclusion and Perspectives synthesizes the main results, discusses academic contributions, acknowledges limitations (e.g., time-homogeneous assumption, single-species focus), and proposes future research directions including time-dependent environmental parameters, multi-species modeling, and development of a user-friendly decision support tool. This thesis demonstrates that the integration of stochastic modeling, exact simulation, numerical optimization, and economic analysis can significantly improve the efficiency and sustainability of scorpion venom production. By moving from a purely empirical to a quantitatively optimized framework, this work provides both theoretical insights and practical tools for researchers and commercial producers alike. The results show that moderate reductions in nutrition (to 88% of ad libitum) combined with precise temperature control (32.6 °C) and an adjusted milking schedule (every 11 days, batch size 6) can increase annual net profit by approximately 6% while keeping the initial investment payback period within a reasonable timeframe (4.8 years). This work opens new avenues for the application of stochastic chemical kinetics to agricultural biotechnology and venom research.

1 LITERATURE REVIEW AND THEORETICAL FRAMEWORK

1.1 Biological Foundations of Scorpion Venom Production

Venom Gland Anatomy and Physiology

Scorpion venom is synthesized and stored in paired venom glands located within the **telson**, the terminal segment of the metasoma (the tail) [5, 1]. The telson is a globular or pear-shaped chitinous capsule that houses these glands, along with the associated musculature and cuticular structures [10, 11].

Anatomically, the venom apparatus consists of a pair of elongated, sac-like venom glands (also called venom sacs) positioned symmetrically within the telson cavity [11]. Each gland is enveloped by a layer of striated muscle fibers. Contraction of these muscles compresses the gland, forcing stored venom through a dedicated, cuticle-lined duct. The two ducts run parallel to each other and open via separate apertures near the tip of the curved, needle-like aculeus (the stinger) [11].

The functional unit of the venom gland is its secretory epithelium. Histological and ultrastructural studies have definitively characterized the mode of secretion in scorpions as **apocrine** [10, 12, 14]. This classification is crucial because it determines what components are present in the extracted venom.

The Apocrine Secretion Mechanism: Unlike merocrine secretion (where toxins are released via exocytosis without loss of cellular material) or holocrine secretion (where the entire cell disintegrates), apocrine secretion is characterized by the budding and pinching-off of the apical portion of the secretory cell's cytoplasm [12, 13]. As illustrated schematically in Figure 1.1.1, this process results in the release of membrane-bound vesicles into the gland lumen. These vesicles contain not only the concentrated venom proteins and peptides but also a payload of cellular components, including portions of the cytosol, cellular organelles, and, most significantly for modern research, **messenger RNA (mRNA) transcripts** [12, 7, 14].

This apocrine mechanism has two major consequences for venom research:

First, the venom is not a pure solution of toxins but a complex suspension of bioactive molecules, salts, and cellular debris [10]. **Second**, and more importantly for transcriptomic studies, because the vesicles bud from the cell, they encapsulate the mRNA that was being actively translated to produce venom components at the time of secretion. This groundbreaking finding, validated by recent transcriptomic research, provides a non-invasive window into the venom gland's genetic activity [7]. As shown in Figure 1.1.2, RNA extracted from venom (e.g., from *Heterometrus laoticus*) can be sequenced, yielding transcriptomes that are remarkably similar to those obtained from the dissected telson tissue itself [7]. This discovery fundamentally changes how we can study venom dynamics, allowing repeated sampling from the same individual over time.

Schematic diagram of apocrine secretion in scorpion venom glands.

The diagram shows a cross-section of the venom gland epithelium with labeled parts: Secretory cell nucleus, Rough Endoplasmic Reticulum (RER), Golgi apparatus, secretory vesicles containing toxins, and the apical cell membrane. The arrow indicates the process of apical cytoplasm budding off, forming a membrane-bound "apocrine bleb" released into the gland lumen containing both venom components and mRNA molecules.

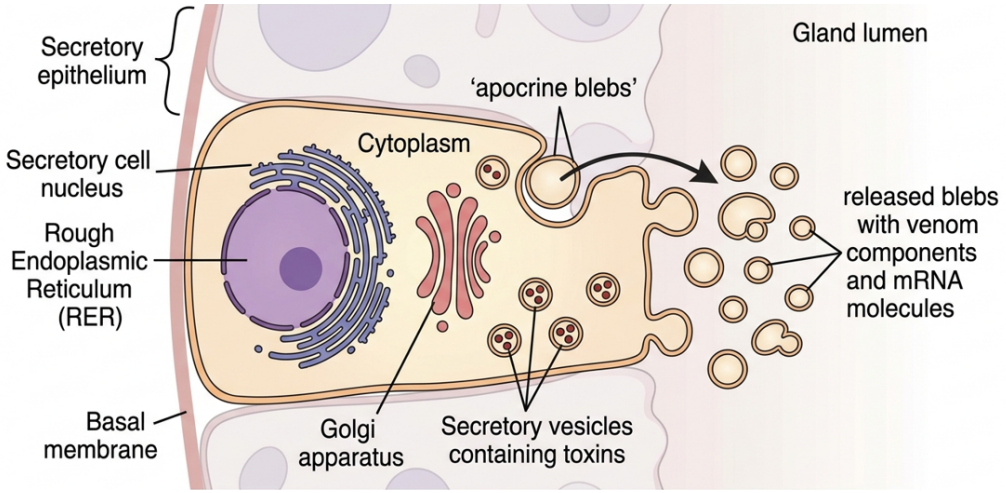


Figure 1.1.1 – Schematic representation of apocrine secretion in scorpion venom glands.

Table 1.1.1 – Comparison of exocrine secretion types in venomous animals.

Feature	**Merocrine Secretion**	**Apocrine Secretion**	**Holocrine Secretion**
Cellular Material Lost	None (exocytosis only)	Apical portion of cytoplasm	Entire cell
mRNA in Secretion	Not typically present	**Present** (encapsulated)	Present (cell lysis)
Example Taxa	Snakes, lizards	**Scorpions, spiders, wasps**	Cone snails, some spiders

Comparison of transcriptomic profiles from telson tissue versus venom.

Left: RIN (RNA Integrity Number) traces showing high-quality RNA from both telson and venom samples. Right: Bar chart comparing relative expression levels (TPM - Transcripts Per Million) of the top 5 most abundant toxin transcripts from both sources, demonstrating high correlation ($R^2 > 0.95$).

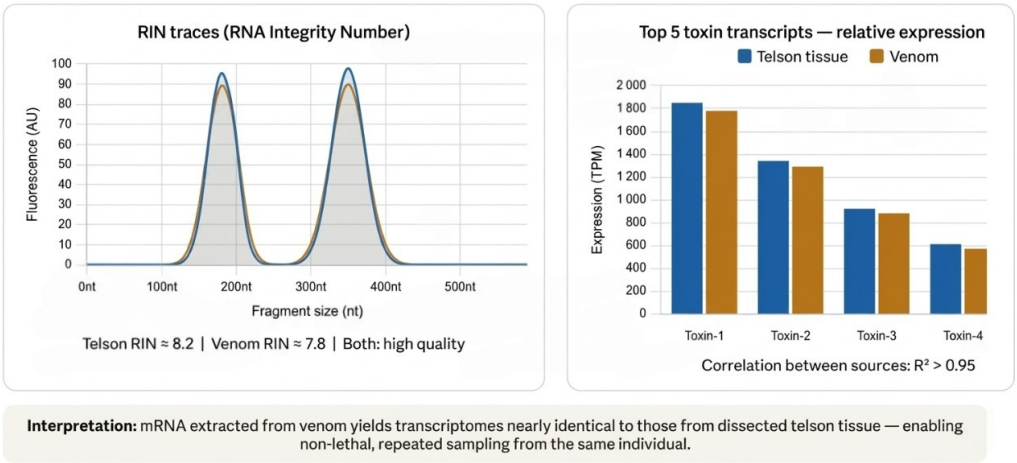


Figure 1.1.2 – Comparison of RNA integrity and transcriptomic profiles from telson tissue vs. venom (adapted from Vonk et al., 2021).

Environmental Influences on Venom Production

The quantity and qualitative composition of scorpion venom are not static; they exhibit significant plasticity in response to a variety of environmental and intrinsic factors [1]. Understanding these modulators is critical for optimizing captive breeding and venom production programs.

Temperature: As ectotherms, scorpions are profoundly influenced by ambient temperature. Their metabolic rate, activity levels, and, consequently, venom regeneration are temperature-dependent [15]. Field and laboratory observations consistently show that scorpion activity, foraging behavior, and venom yield peak during warmer months (summer) when temperatures range from 30°C to 40°C [1]. Conversely, during winter, when temperatures drop to 15–20°C, both activity and venom production decrease significantly [1, 15]. This is directly linked to the temperature coefficient (Q_{10}) of the enzymatic reactions driving venom synthesis. The relationship between temperature and venom production follows a characteristic bell-shaped curve, with an optimal temperature range T_{opt} where production is maximized [1]. The Pearson correlation analysis revealed a significant positive correlation between venom production and temperature, with $r^2 = 0.957$ for *Androctonus finitimus* and $r^2 = 0.955$ for *Hottentotta tamulus* [1].

Diet and Nutrition: Venom production is a metabolically costly process [1]. The availability and quality of dietary nutrients directly impact a scorpion's ability to regen-

erate its venom stores. A controlled feeding study, the results of which are summarized in Table 1.1.2, demonstrated a clear hierarchy in the effect of prey type on venom yield [1]. Scorpions fed a diet of grasshopper nymphs or adults produced the highest volume of venom, followed by those fed house crickets. In contrast, scorpions fed house flies or moths produced the lowest yields. This suggests that prey with a higher nutritional value (e.g., higher protein/lipid content) provides the necessary energetic and biosynthetic precursors for optimal venom gland function [1]. For *Androctonus finitimus*, the venom yield from grasshopper nymphs was 8.8 ± 0.36 μl, which was significantly higher than the yield from house flies (5.5 ± 0.28 μl) with $p < 0.05$ [1].

Table 1.1.2 – Effect of different prey types on venom yield.

Prey Type	**Venom Yield in *A. finitimus* (μl)**	**Venom Yield in *H. tamulus* (μl)**
Grasshopper Nymphs	8.8 ± 0.36	6.0 ± 0.25
Grasshopper Adults	7.4 ± 0.31	4.8 ± 0.26
House Crickets	6.1 ± 0.24	4.0 ± 0.35
Moths	5.1 ± 0.29	2.6 ± 0.36
House Flies	5.5 ± 0.28	2.8 ± 0.39

Source: Adapted from Yaqoob et al. (2016). Values represent mean ± S.D. ($n = 8$ per group).

Milking Frequency and Gland Depletion: Repeated milking (venom extraction) leads to a progressive depletion of the gland's secretory reserves [1]. A study tracking venom yield over consecutive extractions (with 15-day intervals) showed a clear declining trend. For *Androctonus finitimus*, the yield dropped from 9 μl at the first extraction to 4.5 μl by the eighth. For *Hottentotta tamulus*, the yield ceased entirely after the seventh extraction [1]. This indicates that while the gland can rapidly synthesize some components, full replenishment requires a significant recovery period. The observed decline may be due to the exhaustion of specialized secretory cells or physical damage to the gland's epithelial lining from repeated electrical or mechanical stimulation [1].

Geographic and Genetic Variation: The potency and composition of venom are known to vary not only between species but also among populations of the same species from different geographic locations [1, 5]. These variations are attributed to a combination of genetic drift, local adaptation to different prey types, and exposure to distinct environmental pressures (e.g., temperature, humidity). This intra-specific variability has crucial implications for antivenom production, as a monovalent antivenom raised against venom from one region may be less effective against venom from another [16].

Milking Techniques and Their Impacts

The method used to extract venom from a scorpion is a critical variable that directly affects both the yield (quantity) and the biochemical composition (quality) of the obtained product [1, 7]. The two primary methods are manual extraction and electrical stimulation, each with distinct mechanisms and outcomes.

Manual Extraction: This traditional method involves mechanically stimulating the scorpion's telson, often by gently squeezing or tapping it, to induce reflexive venom release. The venom is typically collected in a capillary tube held at the tip of the aculeus [1]. This method yields a very small volume of venom; for example, manual extraction from *Androctonus finitimus* produced only 2.1 ± 0.26 μl [1]. The venom obtained is often referred to as "prevenom" [1]. It is a clear, watery secretion that rapidly changes color (e.g., to blue) upon exposure to air. Proteomic and transcriptomic analyses (e.g., MALDI-TOF MS) reveal that manually extracted venom has a significantly lower molecular diversity. A study found only 5 reproducible peptide mass peaks for *A. finitimus* using this method, compared to 12 peaks for electrically extracted venom [1]. This suggests it may be deficient in many of the high molecular weight, disulfide-rich neurotoxins that are often the most pharmaceutically relevant.

Electrical Stimulation: This is currently the most widely used and efficient method for obtaining high-quality venom in quantities suitable for research and antivenom production [7, 1]. It involves applying a mild, low-voltage electrical current to the scorpion's metasoma, which stimulates the muscles surrounding the venom glands to contract, forcibly ejecting the venom [1, 7]. Typically, a square wave with an amplitude of 18 V and a 10% duty factor at a frequency of 45 Hz is applied through saline-wetted electrodes positioned at the 2nd and 5th metasomal segments [7]. This method yields a significantly higher volume of venom. For the same species, *A. finitimus*, electrical extraction produced 7.6 ± 0.26 μl, which is more than three times the amount obtained manually [1]. Electrically extracted venom is characteristically a dense, opaque white secretion that remains stable in color [1]. Mass spectrometry analysis confirms its superior complexity, with a greater number of peptide peaks across a wider mass-to-charge (m/z) range (2 000–17 000 Da), indicating the presence of a more complete and representative suite of venom components, including the major neurotoxic peptides [1].

Impact on Scorpion Welfare and Sustainability: While electrical stimulation is more productive, it is not without its drawbacks. Repeated milking, even with electricity, can be a source of chronic stress for the scorpion. If the voltage is too high or the stimulation too frequent, it can cause physical damage to the telson muscles or the glandular epithelium itself, leading to the observed decline in yield over time [1, 7]. Therefore, an optimal milking protocol must balance the need for high venom yield with the welfare of the animal to ensure long-term, sustainable production. This includes allowing sufficient recovery periods (e.g., 2–4 weeks) between extractions, providing an optimal diet, and maintaining appropriate environmental conditions (temperature 25–40°C, humidity 50–60%) [1].

A Note on Methodological Advancements: The discovery of significant, high-quality mRNA in scorpion venom, facilitated by electrical extraction, has opened a new frontier. Researchers can now extract RNA directly from the venom itself, as demon-

Table 1.1.3 – Comparative analysis of scorpion venom milking techniques.

Feature	**Manual Extraction**	**Electrical Stimulation**
Principle	Mechanical stimulation of telson	Electrically-induced muscle contraction
Venom Type	"Prevenom" (watery, changes color)	"True venom" (dense, white, stable)
Relative Yield	**Low** ($\sim$ 2.1 μl for *A. finitimus*)	**High** ($\sim$ 7.6 μl for *A. finitimus*)
Molecular Diversity (MALDI-TOF)	**Low** (5 peaks for *A. finitimus*)	**High** (12 peaks for *A. finitimus*)
Primary Use	Historical studies, minor applications	Research, antivenom production, industry
Animal Welfare	Variable, potential for injury	Less stressful if protocol optimized

Source: Yaqoob et al. (2016) and Vonk et al. (2021).

strated with *Heterometrus laoticus*, to generate "venom-derived transcriptomes" [7]. This non-lethal approach allows for the longitudinal study of gene expression within a single individual over time, enabling investigations into how diet, age, season, or milking frequency dynamically alter the venom's genetic blueprint without sacrificing the animal [7]. This represents a major step forward in ethical and sophisticated venom research.

1.2 Previous Mathematical Models in Biological Production Systems

Mathematical modeling of biological production systems has evolved significantly over the past century, progressing from simple deterministic descriptions to sophisticated stochastic frameworks that capture the inherent randomness of biological processes. This section reviews the major modeling paradigms that have been developed, with particular emphasis on their applicability to venom production systems and their relevance to the stochastic birth-death framework developed in this thesis.

Deterministic and Age-Structured Models

The earliest mathematical descriptions of biological production systems were deterministic in nature, treating populations as continuous variables that evolve according to ordinary differential equations (ODEs). This approach, rooted in classical chemical kinetics, assumes that molecular populations are sufficiently large that random fluctuations can be neglected [3, 4].

Exponential and Logistic Growth Models: The simplest deterministic model for a production system is the exponential growth model, which assumes that the rate of

Schematic overview of venom extraction methods.

Left: Traditional method requiring sacrifice of the animal to extract the venom gland from the telson. Right: New non-lethal method using RNA extraction from venom obtained by electrostimulation, allowing multiple time-point sampling from the same individual.

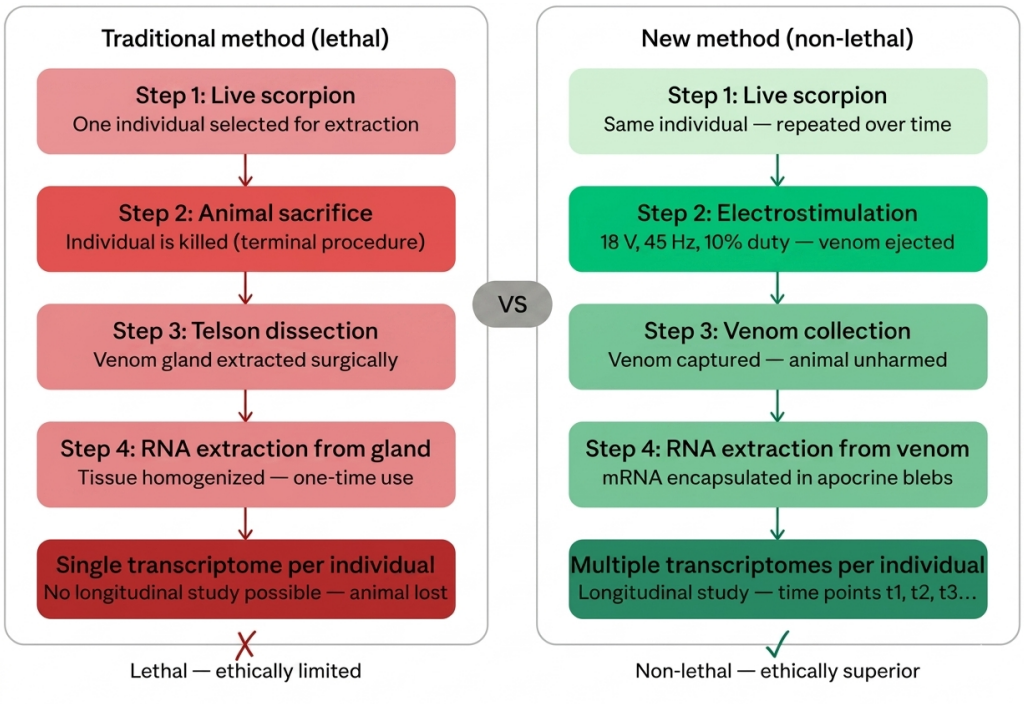


Figure 1.1.3 – Comparison of traditional and non-lethal methods for venom gland transcriptome generation (adapted from Vonk et al., 2021).

increase is proportional to the current quantity. In the context of venom production, if we denote by $x(t)$ the quantity of venom at time t, the exponential growth model takes the form:

$$\frac{dx(t)}{dt} = r\,x(t), \tag{1.2.1}$$

where $r = \lambda - \mu$ is the net growth rate, with λ representing the production rate and μ the degradation rate. The solution to this equation is:

$$x(t) = x_0 e^{rt}, \tag{1.2.2}$$

where $x_0 = x(0)$ is the initial venom quantity. While this model captures the basic idea of production and loss, it has significant limitations. As noted by Gillespie (2007), the deterministic solution depends only on the difference $\lambda - \mu$ and not on the individual values of λ and μ, which has important implications for parameter identifiability [3].

A more realistic deterministic model incorporates carrying capacity, leading to the

logistic growth equation:

$$\frac{dx(t)}{dt} = r\,x(t)\left(1 - \frac{x(t)}{K}\right), \tag{1.2.3}$$

where K is the maximum sustainable venom quantity (carrying capacity). Note that K is used only in this introductory deterministic discussion and is unrelated to the half-saturation constant K_N introduced in Chapter 2 (Eq. (2.2.3)). The logistic model has the analytical solution:

$$x(t) = \frac{Kx_0e^{rt}}{K + x_0(e^{rt} - 1)}. \tag{1.2.4}$$

In the context of venom glands, the carrying capacity K would represent the maximum storage capacity of the gland, which is determined by anatomical constraints [10]. However, these deterministic models fail to capture the discrete and stochastic nature of molecular interactions, which become critically important when dealing with small molecular populations [4].

Comparison of exponential and logistic growth models.

The figure shows typical trajectories of the exponential growth model (dashed line) and logistic growth model (solid line) for the same initial condition $x_0 = 1$ and growth rate $r = 0.1$, with carrying capacity $K = 100$ for the logistic model.

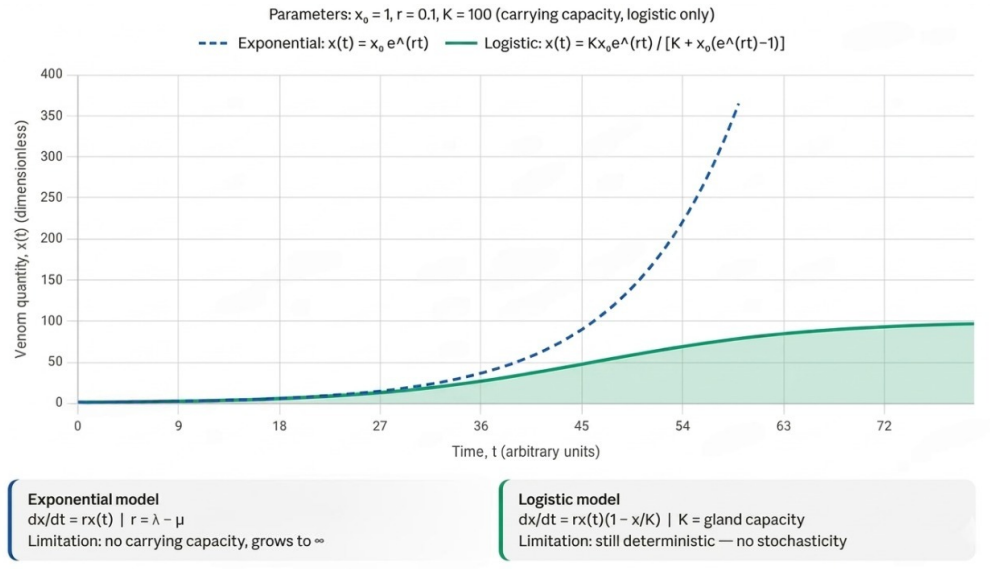


Figure 1.2.1 – Comparison of exponential and logistic deterministic growth models.

Age-Structured Models: The McKendrick–Von Foerster Equation: A more sophisticated deterministic approach incorporates age structure into the population description. The McKendrick–Von Foerster equation, originally developed for age-structured populations, provides a framework for modeling systems where individuals

(or molecules) have different ages that affect their behavior [2]. For venom production, one could consider the age of venom molecules within the gland, as newly synthesized molecules may have different properties than aged ones.

The McKendrick–Von Foerster equation takes the form:

$$\frac{\partial n(a,t)}{\partial t} + \frac{\partial n(a,t)}{\partial a} = -\mu(a)\, n(a,t), \tag{1.2.5}$$

where $n(a,t)$ is the density of molecules of age a at time t, and $\mu(a)$ is the age-dependent degradation rate. The boundary condition at age $a = 0$ is given by:

$$n(0,t) = \int_0^\infty \beta(a)\, n(a,t)\, da, \tag{1.2.6}$$

where $\beta(a)$ is the age-specific production rate. While this approach is powerful for certain biological systems, it introduces significant mathematical complexity and requires detailed knowledge of age-dependent parameters that may not be available for venom production systems [17].

Stochastic and Lattice-Based Models

The limitations of deterministic approaches have led to the development of stochastic models that explicitly account for the discrete and random nature of molecular interactions. These models are particularly relevant for venom production, where the numbers of certain molecules can be very small [4, 3].

Birth-Death Processes: The birth-death process is the fundamental stochastic framework for modeling populations that change through individual birth and death events. In the context of venom production, we can view the synthesis of new venom molecules as "births" and their degradation or removal as "deaths" [2, 4].

Let $X(t)$ denote the number of venom molecules at time t. The birth-death process is characterized by transition rates:

$$\begin{aligned} \mathbb{P}(X(t+dt) = i+1 \mid X(t) = i) &= \lambda_i\, dt + o(dt), \\ \mathbb{P}(X(t+dt) = i-1 \mid X(t) = i) &= \mu_i\, dt + o(dt), \end{aligned} \tag{1.2.7}$$

where λ_i and μ_i are the state-dependent birth and death rates, respectively. In the simplest case of constant rates ($\lambda_i = \lambda i$, $\mu_i = \mu i$), the process is known as the linear birth-death process [3]. The probability generating function for this process satisfies:

$$\frac{\partial G(z,t)}{\partial t} = (z-1)\,(\lambda z - \mu)\,\frac{\partial G(z,t)}{\partial z}, \tag{1.2.8}$$

where $G(z,t) = \mathbb{E}[z^{X(t)}]$. This equation can be solved analytically. For the critical case $\lambda = \mu$, the probability distribution is given by (Kendall, 1948; Bailey, 1964):

$$\mathbb{P}(X(t) = i \mid X(0) = i_0) = \begin{cases} \binom{i-1}{i_0-1} \left(\frac{\lambda t}{1+\lambda t}\right)^{i-i_0} \left(\frac{1}{1+\lambda t}\right)^{i_0}, & i \geq i_0 \geq 1, \\ \left(\frac{\lambda t}{1+\lambda t}\right)^{i_0}, & i = 0, \end{cases} \tag{1.2.9}$$

or equivalently $\mathbb{P}(X(t) = i) = \binom{i-1}{i_0-1}(e^{-\mu t})^{i_0}(1-e^{-\mu t})^{i-i_0}$ when $\lambda = \mu$, with the understanding that $1 - e^{-\mu t} = \mu t/(1+\mu t)$. For $\lambda \neq \mu$, the distribution can be expressed in terms of negative binomial probabilities [2].

Comparison of deterministic and stochastic birth-death trajectories.

The figure shows five realizations of a stochastic linear birth-death process (jagged lines) together with the continuous deterministic solution (smooth line). Parameters: $\lambda = 3$, $\mu = 4$, $x_0 = 50$. The stochastic realizations exhibit extinction events and considerable variability not captured by the deterministic model.

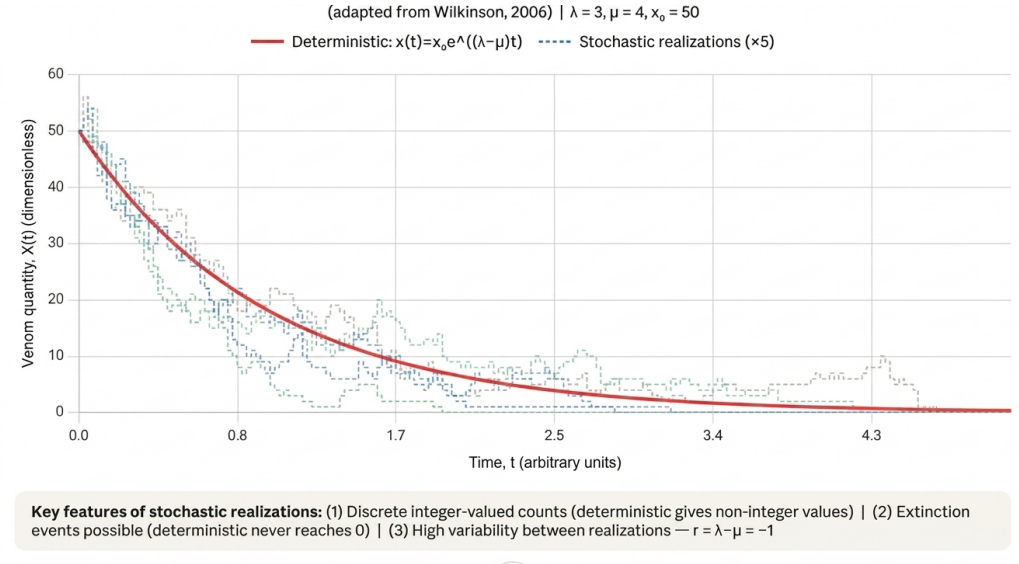


Figure 1.2.2 – Comparison of deterministic and stochastic birth-death process trajectories (adapted from Wilkinson, 2006).

As demonstrated by Wilkinson (2006), the deterministic solution $x(t) = x_0 e^{(\lambda-\mu)t}$ fails to capture several important features of the stochastic process, including:

1. The discrete nature of molecule counts (the deterministic solution often gives non-integer values);

2. The possibility of extinction (the deterministic solution approaches zero but never reaches it);

3. The variability between different realizations of the process [4].

Density-Dependent Stochastic Models: In many biological systems, including venom production, the rates of birth and death events depend on the current population density. For venom production, the production rate may be limited by the availability of precursors or by feedback mechanisms that regulate gland activity [3].

A general density-dependent birth-death process has transition rates of the form:

$$\lambda_i = i \cdot f\left(\frac{i}{V}\right), \quad \mu_i = i \cdot g\left(\frac{i}{V}\right), \tag{1.2.10}$$

where V is the system volume and f and g are functions describing the per-capita rates. For such processes, the deterministic approximation is given by:

$$\frac{dx}{dt} = x\, f\left(\frac{x}{V}\right) - x\, g\left(\frac{x}{V}\right), \tag{1.2.11}$$

which is known as the logistic equation when $f(x/V) = \lambda$ and $g(x/V) = \mu + \gamma x/V$ [2].

The chemical master equation (CME) provides the most complete description of density-dependent stochastic systems. For a system with M reaction channels, the CME takes the form [3, 4]:

$$\frac{\partial P(\mathbf{x}, t \mid \mathbf{x}_0, t_0)}{\partial t} = \sum_{j=1}^{M} \left[a_j(\mathbf{x} - \boldsymbol{\nu}_j) P(\mathbf{x} - \boldsymbol{\nu}_j, t \mid \mathbf{x}_0, t_0) - a_j(\mathbf{x}) P(\mathbf{x}, t \mid \mathbf{x}_0, t_0)\right], \tag{1.2.12}$$

where $a_j(\mathbf{x})$ is the propensity function for reaction j, and $\boldsymbol{\nu}_j$ is the state-change vector [3]. For the simple birth-death process, the CME reduces to:

$$\frac{\partial P_i(t)}{\partial t} = \lambda_{i-1} P_{i-1}(t) + \mu_{i+1} P_{i+1}(t) - (\lambda_i + \mu_i) P_i(t). \tag{1.2.13}$$

Lattice-Based Models and Exclusion Processes: For systems where spatial organization plays an important role, lattice-based models provide a framework for incorporating spatial structure into stochastic descriptions [17]. In these models, the state space is discretized into a lattice, and particles (molecules) can move between lattice sites and interact with neighboring particles.

The simplest lattice-based model is the simple exclusion process (SEP), where particles hop to adjacent empty sites with a given rate [2]. For venom production, such models could capture the spatial distribution of venom molecules within the gland and the effects of molecular crowding on production rates.

The master equation for the exclusion process on a one-dimensional lattice of L sites is:

$$\frac{d}{dt}\mathbb{P}(\eta,t) = \sum_{x=1}^{L-1} \left[\mathbb{P}(\eta^{x,x+1},t) - \mathbb{P}(\eta,t)\right] \mathbf{1}_{\eta(x)\neq\eta(x+1)}, \tag{1.2.14}$$

where $\eta(x) \in \{0,1\}$ indicates whether site x is occupied, and $\eta^{x,x+1}$ is the configuration obtained by swapping the occupation numbers at sites x and $x+1$ [2].

More sophisticated reaction-diffusion models combine movement with chemical reactions. For a system with diffusion and birth-death kinetics, the stochastic dynamics are described by:

$$\frac{\partial u(\mathbf{r},t)}{\partial t} = D\nabla^2 u(\mathbf{r},t) + \lambda(\mathbf{r},t)u(\mathbf{r},t) - \mu(\mathbf{r},t)u(\mathbf{r},t), \tag{1.2.15}$$

where $u(\mathbf{r},t)$ is the local density at position $\mathbf{r}$ and time t, and D is the diffusion coefficient [17]. Note that $u(\mathbf{r},t)$ denotes a spatial density and should not be confused with the traffic intensity $\rho = \lambda/\mu$ used elsewhere in this thesis (Section 2.3.1). While these spatial models provide more detailed descriptions, they also introduce significant computational complexity and require parameter values that may be difficult to obtain experimentally.

Table 1.2.1 – Summary of mathematical models for biological production systems.

Model Type	**Key Equations**	**Advantages**	**Limitations**
Exponential Growth	$dx/dt = rx$	Simple, analytical solution	No carrying capacity
Logistic Growth	$dx/dt = rx(1 - x/K)$	Includes saturation	Deterministic only
Age-Structured	$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a} = -\mu(a)n$	Captures age effects	Complex, requires age data
Birth-Death (CTMC)	CME: $\partial_t P = \sum[\lambda P_{i-1} + \mu P_{i+1} - (\lambda+\mu)P_i]$	Discrete, stochastic	Computationally intensive
Lattice-Based	Exclusion processes, reaction-diffusion	Captures spatial effects	Very complex, many parameters

Application to Venom Production: The stochastic birth-death framework is particularly well-suited for modeling venom production for several reasons. First, venom molecules are produced and degraded in discrete quantities, making the discrete nature of the process essential [7]. Second, the numbers of certain venom components can be very small, especially during the early stages of gland replenishment, making stochastic effects significant [3]. Third, the production process is subject to intrinsic randomness due

to the probabilistic nature of molecular interactions [4]. Finally, the apocrine secretion mechanism, which releases cytoplasmic contents including mRNA, introduces additional stochasticity into the process [7].

The model developed in this thesis extends the basic birth-death framework by incorporating:

1. Temperature-dependent production and degradation rates;
2. Nutrition-dependent production rates;
3. Milking events as discrete removal processes;
4. State-dependent rates that capture feedback mechanisms.

These extensions are necessary to capture the full complexity of scorpion venom production and to provide a realistic basis for optimization of commercial production.

Schematic representation of model hierarchy for biological production systems.

The diagram shows the relationships between different modeling approaches: from deterministic ODE models at the top, through age-structured models, to fully stochastic CTMC models, and finally to spatial lattice-based models. Arrows indicate increasing complexity and biological realism.

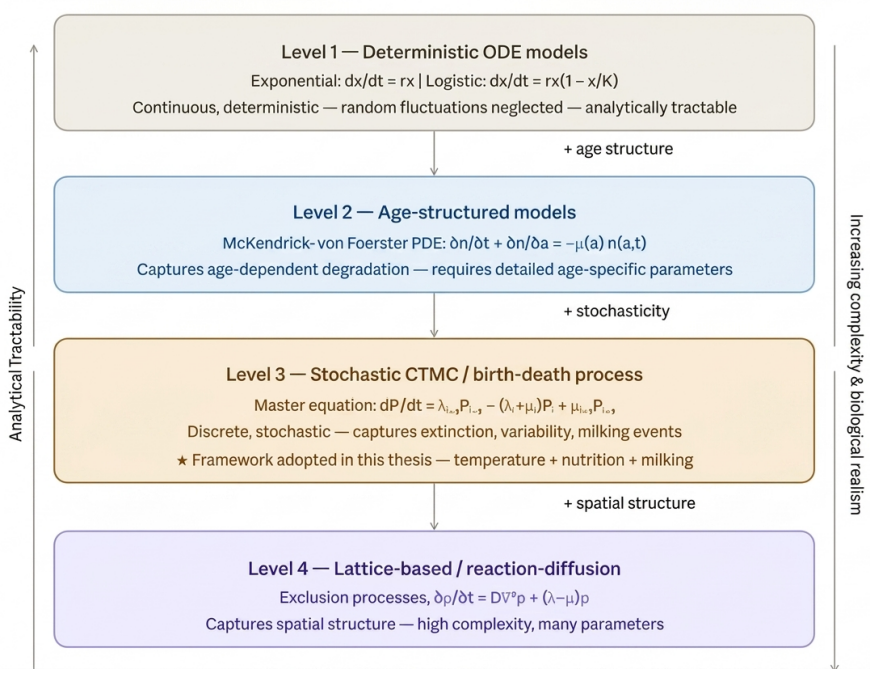


Figure 1.2.3 – Hierarchy of mathematical models for biological production systems.

The choice of a birth-death process as the foundation for our venom production model is justified by its ability to capture the essential discrete and stochastic nature of the system while remaining analytically tractable [2, 4]. Moreover, the birth-death framework

provides natural connections to both deterministic approximations (through the law of large numbers) and diffusion approximations (through the central limit theorem), allowing for multi-scale analysis of the system [3].

1.3 Mathematical Foundations

This section establishes the rigorous mathematical framework necessary for the stochastic modeling of scorpion venom production. We build upon the biological foundations laid in Section 1.1 and the review of existing models in Section 1.2. The core concepts introduced here—continuous-time Markov chains, Kolmogorov equations, the Gillespie algorithm, and moment closure techniques—will be directly applied in Chapter 2 to formulate our novel stochastic model of venom synthesis and degradation.

1.3.1 Continuous-Time Markov Chains (CTMC)

A **continuous-time Markov chain** is a stochastic process $\{X(t), t \geq 0\}$ taking values in a countable state space S (here, $S = \mathbb{N}_0 = \{0, 1, 2, \dots\}$ representing the number of venom molecules) that possesses the **Markov property**: the future evolution depends only on the present state, not on the past. Formally, for any $t_0 < t_1 < \cdots < t_n < t_{n+1}$ and any states $i_0, \dots, i_{n+1} \in S$,

$$\mathbb{P}\Big(X(t_{n+1}) = i_{n+1} \mid X(t_0) = i_0, \dots, X(t_n) = i_n\Big) = \mathbb{P}\Big(X(t_{n+1}) = i_{n+1} \mid X(t_n) = i_n\Big). \tag{1.3.1}$$

This memoryless property is the cornerstone of analytical tractability.

Definition 1.1 (Transition probabilities)**.** *For a time-homogeneous CTMC, the transition probability from state i to state j in time t is denoted*

$$p_{ij}(t) = \mathbb{P}\Big(X(t+s) = j \mid X(s) = i\Big), \tag{1.3.2}$$

*which does not depend on s. The matrix $\mathbf{P}(t) = (p_{ij}(t))_{i,j \in S}$ is called the **transition semigroup** and satisfies $\mathbf{P}(0) = \mathbf{I}$ (identity matrix) and the Chapman–Kolmogorov equations $\mathbf{P}(s+t) = \mathbf{P}(s)\mathbf{P}(t)$.*

Definition 1.2 (Generator matrix or Q-matrix)**.** *The **infinitesimal generator** (or **Q-matrix**) is defined by*

$$q_{ij} = \lim_{\Delta t \to 0^+} \frac{p_{ij}(\Delta t) - \delta_{ij}}{\Delta t}, \qquad i, j \in S, \tag{1.3.3}$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. The matrix $\mathbf{Q} = (q_{ij})$ satisfies:

$$q_{ii} \leq 0, \quad q_{ij} \geq 0 \quad (i \neq j), \tag{1.3.4}$$

$$\sum_{j \in S} q_{ij} = 0 \quad \forall i. \tag{1.3.5}$$

*The quantity $q_i = -q_{ii}$ is the **total exit rate** from state i.*

For a birth–death process relevant to venom production, the only non-zero off-diagonal entries are:

$$q_{i,i+1} = \lambda_i \quad \text{(birth rate)}, \qquad q_{i,i-1} = \mu_i \quad \text{(death rate)}, \tag{1.3.6}$$

with $\lambda_i, \mu_i \geq 0$ and $q_{ii} = -(\lambda_i + \mu_i)$.

Theorem 1.1 (Characterization of CTMC via Q-matrix)**.** *Let $\mathbf{Q}$ be a matrix satisfying the above conditions and assume $\sup_i q_i < \infty$ (uniformly bounded rates). Then there exists a unique CTMC with generator $\mathbf{Q}$, and the transition probabilities satisfy the **backward** and **forward** differential equations:*

$$\frac{d}{dt}\mathbf{P}(t) = \mathbf{Q}\mathbf{P}(t) \quad (\textit{backward}), \tag{1.3.7}$$

$$\frac{d}{dt}\mathbf{P}(t) = \mathbf{P}(t)\mathbf{Q} \quad (\textit{forward}), \tag{1.3.8}$$

with initial condition $\mathbf{P}(0) = \mathbf{I}$.

Proof sketch. The existence follows from the construction via the jump chain and holding times. For bounded rates, the matrix exponential $\mathbf{P}(t) = \exp(\mathbf{Q}t)$ converges and satisfies both equations uniquely. See [2], Chapter 2 for details. □

1.3.2 Kolmogorov Differential Equations (Forward Equation)

The **Kolmogorov forward equation** (also called the **master equation** in chemical kinetics) governs the time evolution of the probability distribution $p_i(t) = \mathbb{P}(X(t) = i)$. It is derived from the Chapman–Kolmogorov equation by considering an infinitesimal time increment.

Theorem 1.2 (Kolmogorov forward equation)**.** *For a CTMC with generator $\mathbf{Q}$, the state probabilities $p_i(t)$ satisfy the system of ordinary differential equations:*

$$\frac{dp_i(t)}{dt} = \sum_{j \neq i} \Big(p_j(t) q_{ji} - p_i(t) q_{ij} \Big), \qquad i \in S. \tag{1.3.9}$$

Equivalently, in vector form $\mathbf{p}(t) = (p_i(t))_{i \in S}$,

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{p}(t)\mathbf{Q}. \tag{1.3.10}$$

Proof. From the Chapman–Kolmogorov equation,

$$p_i(t + \Delta t) = \sum_j p_j(t) p_{ji}(\Delta t).$$

Expanding $p_{ji}(\Delta t) = \delta_{ji} + q_{ji}\Delta t + o(\Delta t)$ and using $\sum_j p_j(t) = 1$, we obtain

$$\frac{p_i(t + \Delta t) - p_i(t)}{\Delta t} = \sum_{j \neq i} p_j(t) q_{ji} - p_i(t) \sum_{j \neq i} q_{ij} + o(1).$$

Taking $\Delta t \to 0$ and using $q_{ii} = -\sum_{j \neq i} q_{ij}$ gives the result. □

For a **birth–death process** (the foundation of our venom model), the forward equations simplify to:

$$\frac{dp_i}{dt} = \lambda_{i-1} p_{i-1} - (\lambda_i + \mu_i) p_i + \mu_{i+1} p_{i+1}, \quad i \geq 1, \tag{1.3.11}$$

with $\lambda_{-1} = 0$ and a special equation for $i = 0$:

$$\frac{dp_0}{dt} = -\lambda_0 p_0 + \mu_1 p_1. \tag{1.3.12}$$

These equations are exact but generally infinite-dimensional. They form the starting point for both analytical (when solvable) and numerical (when using simulation) approaches.

1.4 The Chapman–Kolmogorov Equation and the Master Equation

1.4.1 Derivation from the Chapman–Kolmogorov Equation

By differentiating the Chapman–Kolmogorov relation $\mathbf{P}(s + t) = \mathbf{P}(s)\,\mathbf{P}(t)$ with respect to t at $t = 0$, one obtains the *Kolmogorov equations*:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) = \mathbf{P}(t)\,\mathbf{Q} \quad \text{(forward)}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) = \mathbf{Q}\,\mathbf{P}(t) \quad \text{(backward)}.$$

In scalar notation, the forward equation gives the (i, j) component:

Theorem 1.3 (Master equation of the birth-death process)**.** *Let* $p_j(t) = \mathbb{P}(X_t = j \mid X_0 =$

i) (marginal law). Then, for $j \geq 1$:

$$\frac{\mathrm{d}p_j}{\mathrm{d}t}(t) = \lambda_{j-1}\, p_{j-1}(t) - (\lambda_j + \mu_j)\, p_j(t) + \mu_{j+1}\, p_{j+1}(t), \tag{1.4.1}$$

and for $j = 0$:

$$\frac{\mathrm{d}p_0}{\mathrm{d}t}(t) = -\lambda_0\, p_0(t) + \mu_1\, p_1(t).$$

Proof. Applying the Chapman–Kolmogorov equation with a step $\mathrm{d}t \to 0$:

$$\begin{aligned} p_j(t+\mathrm{d}t) &= \sum_k p_k(t)\, q_{kj}\, \mathrm{d}t + p_j(t)(1 - (\lambda_j + \mu_j)\mathrm{d}t) + O(\mathrm{d}t^2) \\ &= p_{j-1}(t)\, \lambda_{j-1}\, \mathrm{d}t + p_j(t)\Big(1 - (\lambda_j + \mu_j)\mathrm{d}t\Big) + p_{j+1}(t)\, \mu_{j+1}\, \mathrm{d}t + O(\mathrm{d}t^2). \end{aligned}$$

Subtracting $p_j(t)$ and dividing by $\mathrm{d}t$, we obtain equation (1.4.1). □

1.4.2 Matrix Form

The master equation reads $\dot{\boldsymbol{p}}(t) = \boldsymbol{p}(t)\, Q$, where $\boldsymbol{p}(t)$ is the row vector of probabilities and Q is the tridiagonal matrix:

$$Q = \begin{pmatrix} -\lambda_0 & \lambda_0 & 0 & 0 & \cdots \\ \mu_1 & -(\mu_1+\lambda_1) & \lambda_1 & 0 & \cdots \\ 0 & \mu_2 & -(\mu_2+\lambda_2) & \lambda_2 & \cdots \\ 0 & 0 & \mu_3 & -(\mu_3+\lambda_3) & \cdots \\ \vdots & & & & \ddots \end{pmatrix}. \tag{1.4.2}$$

The formal solution is $\boldsymbol{p}(t) = \boldsymbol{p}(0)\, e^{Qt}$, given directly by the Chapman–Kolmogorov semigroup.

Theorem 1.4 (Stationary distribution)**.** *If the CTMC is irreducible and positive recurrent, there exists a unique stationary distribution $\boldsymbol{\pi} = (\pi_i)_{i \in S}$ satisfying $\pi_i \geq 0$, $\sum_i \pi_i = 1$, and*

$$\pi_j = \sum_i \pi_i p_{ij}(t) \quad \forall t, \qquad \textit{equivalently } \boldsymbol{\pi} \mathbf{Q} = \mathbf{0}. \tag{1.4.3}$$

For a birth–death process, the stationary distribution (if it exists) is given by the detailed balance equations:

$$\pi_i \lambda_i = \pi_{i+1} \mu_{i+1}, \quad i \geq 0, \tag{1.4.4}$$

leading to

$$\pi_i = \pi_0 \prod_{k=0}^{i-1} \frac{\lambda_k}{\mu_{k+1}}, \qquad \pi_0^{-1} = 1 + \sum_{i=1}^{\infty} \prod_{k=0}^{i-1} \frac{\lambda_k}{\mu_{k+1}}. \tag{1.4.5}$$

Proof. Detailed balance is a sufficient condition for reversibility and stationarity (see [2], Theorem 1.9.3). □

In our venom production model with milking, the detailed balance is broken because the milking event (jump from i to $i - m$) is not reversible. Hence we must solve the full forward equation or compute the stationary distribution via the method of characteristics or generating functions (see Chapter 2).

1.4.3 The Gillespie Algorithm (Exact Stochastic Simulation)

When analytical solution of the master equation is impossible (as is the case for most realistic biological models, including our venom production system with state-dependent rates), we resort to **exact stochastic simulation**. The most widely used method is the **Gillespie algorithm** (also called the **stochastic simulation algorithm – SSA**), which generates sample paths (trajectories) of the CTMC without approximation.

Derivation from first principles

Consider a CTMC with M possible reaction channels (transitions). Let $a_j(\mathbf{x})$ be the **propensity function** of channel j, defined such that

$$a_j(\mathbf{x})\,dt = \mathbb{P}\Big(\text{one } R_j \text{ reaction occurs in } (t, t+dt] \mid X(t) = \mathbf{x}\Big). \tag{1.4.6}$$

The total propensity is $a_0(\mathbf{x}) = \sum_{j=1}^{M} a_j(\mathbf{x})$.

Theorem 1.5 (Joint distribution of next reaction time and type)**.** *Given the system is in state* $\mathbf{x}$ *at time* t*, the time* τ *to the next reaction is exponentially distributed with rate* $a_0(\mathbf{x})$*, and the index* j *of the next reaction is independent of* τ *with probability* $a_j(\mathbf{x})/a_0(\mathbf{x})$*. Formally,*

$$p(\tau, j \mid \mathbf{x}) = a_j(\mathbf{x}) \exp\Big(-a_0(\mathbf{x})\tau\Big). \tag{1.4.7}$$

Proof. The probability that no reaction occurs in $[t, t+\tau)$ is $\exp(-a_0(\mathbf{x})\tau)$ (since the waiting time is memoryless). The probability that the next reaction is of type j and occurs in $[t+\tau, t+\tau+d\tau)$ equals

$$\Big(\text{probability of no reaction in } [t, t+\tau)\Big) \times \Big(a_j(\mathbf{x})\,d\tau\Big) = e^{-a_0(\mathbf{x})\tau} a_j(\mathbf{x})\,d\tau.$$

The factorisation shows independence. □

The direct method (Gillespie, 1977)

1. **Initialise** $t = t_0$, state $\mathbf{x} = \mathbf{x}_0$.

2. **Compute** all propensities $a_j(\mathbf{x})$ and their sum $a_0(\mathbf{x})$.

3. **Generate** two independent uniform random numbers $r_1, r_2 \sim U(0,1)$.

4. **Set** $\tau = \frac{1}{a_0(\mathbf{x})} \ln\left(\frac{1}{r_1}\right)$.

5. **Find** the smallest integer j such that $\sum_{k=1}^{j} a_k(\mathbf{x}) > r_2 a_0(\mathbf{x})$.

6. **Update** $t \leftarrow t + \tau$, $\mathbf{x} \leftarrow \mathbf{x} + \boldsymbol{\nu}_j$ (where $\boldsymbol{\nu}_j$ is the state-change vector for reaction j).

7. **Record** $(t, \mathbf{x})$ as desired.

8. **Return** to step 2 or stop.

Remark 1.1. *The algorithm is **exact** in the sense that it generates realizations of the true stochastic process (the CTMC) with no numerical approximation error aside from machine precision and pseudo-random number quality.*

Table 1.4.1 – Comparison of direct method and first-reaction method.

Feature	**Direct Method**	**First-Reaction Method**
Random numbers per step	2	M
Computational cost per step	$O(M)$ (to find j)	$O(M)$ (to find minimum)
Efficiency for large M	Good (with optimization)	Poor (many exponentials)
Typical use	Standard Gillespie SSA	Conceptual basis for Gibson–Bruck

For our venom production model (Chapter 3), we will implement an optimised version of the direct method with the following propensity functions:

— **Production (birth):** $a_{\text{prod}}(i) = \lambda_i = f(T) \cdot g(N_u) \cdot i$

— **Degradation (death):** $a_{\text{deg}}(i) = \mu_i = h(T) \cdot i$

— **Milking:** $a_{\text{milk}}(i) = \gamma \cdot \mathbf{1}_{i \geq m}$.

The state-change vectors are $\boldsymbol{\nu}_{\text{prod}} = +1$, $\boldsymbol{\nu}_{\text{deg}} = -1$, and $\boldsymbol{\nu}_{\text{milk}} = -m$.

1.4.4 Moment Closure and Analytical Tools

Even with the Gillespie algorithm, we often need **analytical approximations** to understand the mean and variance of the process without running thousands of simulations. This is where **moment equations** and **closure approximations** become essential.

Moment equations from the master equation

Multiplying the master equation by i and summing over all i yields the equation for the first moment $\mathbb{E}[X(t)] = \sum_i i p_i(t)$:

$$\frac{d\mathbb{E}[X]}{dt} = \mathbb{E}\big[\lambda(X) - \mu(X)\big]. \tag{1.4.8}$$

For linear birth–death rates ($\lambda_i = \lambda i$, $\mu_i = \mu i$), this closes immediately:

$$\frac{d\mathbb{E}[X]}{dt} = (\lambda - \mu)\mathbb{E}[X]. \tag{1.4.9}$$

For **nonlinear rates** (e.g., $\lambda_i = \lambda i$ but with milking), the equation involves higher moments. The second moment equation involves third moments, and so on – this is the **moment closure problem**.

Cumulant generating function and closure approximations

A powerful tool is the **cumulant generating function** $K(\theta, t) = \ln \mathbb{E}[e^{\theta X(t)}]$. Expanding in powers of θ:

$$K(\theta, t) = \sum_{r=1}^{\infty} \kappa_r(t) \frac{\theta^r}{r!}, \tag{1.4.10}$$

where $\kappa_1 = \mathbb{E}[X]$ (mean), $\kappa_2 = \mathrm{Var}(X)$ (variance), κ_3 (skewness), etc.

Normal (Gaussian) closure: Assume all cumulants of order ≥ 3 are zero. Then the distribution is fully characterized by mean $m(t)$ and variance $v(t)$. The closure approximation yields:

$$\frac{dm}{dt} = \mathbb{E}[\lambda(X) - \mu(X)] \approx \lambda(m) - \mu(m) \quad \text{(if rates are linear in } X\text{)}, \tag{1.4.11}$$

and for variance:

$$\frac{dv}{dt} = 2\Big(\lambda'(m) - \mu'(m)\Big)v + \lambda(m) + \mu(m). \tag{1.4.12}$$

For our venom production model with $\lambda_i = \lambda i$ and $\mu_i = \mu i$, the mean equation is exact, and the variance equation becomes:

$$\frac{dv}{dt} = 2(\lambda - \mu)v + (\lambda + \mu)\,\mathbb{E}[X]. \tag{1.4.13}$$

This will be derived in Chapter 2.

Generating function method

For discrete-state processes, the **probability generating function** (PGF) $G(z, t) = \sum_{i=0}^{\infty} p_i(t) z^i$ often converts the master equation into a partial differential equation that can sometimes be solved by the method of characteristics. For the simple birth–death process, we have:

$$\frac{\partial G}{\partial t} = (\lambda z - \mu)(z - 1)\frac{\partial G}{\partial z}, \tag{1.4.14}$$

with solution

$$G(z, t) = \left(\frac{\mu(z-1)e^{(\lambda-\mu)t} - \lambda z + \mu}{\lambda(z-1)e^{(\lambda-\mu)t} - \lambda z + \mu}\right)^a$$

for $X(0) = a$. The PGF method is used in Chapter 2 to obtain the stationary distribution of our venom model with milking.

1.5 Chapter Summary

Chapter 1 has laid the comprehensive groundwork for the stochastic modeling of scorpion venom production. We began with the **biological foundations** (Section 1.1), describing the anatomy of the venom gland, the apocrine secretion mechanism that releases mRNA into the venom, and the environmental factors (temperature, nutrition, milking frequency) that modulate venom yield. This biological understanding directly motivates the mathematical structures developed later.

In **Section 1.2**, we reviewed existing mathematical models for biological production systems, ranging from simple deterministic exponential and logistic growth to age-structured McKendrick–von Foerster equations, and from stochastic birth–death processes to lattice-based exclusion models. We identified a critical gap: no existing model integrates
temperature-dependent birth rates, nutrition-dependent saturation, and discrete milking events in a unified stochastic framework. This gap justifies the original contribution of our thesis.

Section 1.3 introduced the rigorous mathematical toolkit necessary for our work:

— **Continuous-time Markov chains (CTMC)** provide the probabilistic foundation, with the generator matrix $\mathbf{Q}$ encapsulating all transition rates.

— **Kolmogorov forward equations** (master equation) govern the time evolution of state probabilities; for birth–death processes they reduce to a tractable infinite system.

— The **Gillespie algorithm** offers an exact simulation method, generating sample paths by drawing exponential waiting times and selecting reaction events according to their propensities. This will be our primary computational engine in Chapter 3.

— **Moment closure techniques** and generating functions provide analytical approximations for mean and variance, which we will use to derive optimal milking schedules in Chapter 4.

The synergy between these mathematical tools and the biological reality of scorpion venom production is now clear: the discrete, random nature of molecular synthesis and degradation is naturally captured by a CTMC; the environmental influences (temperature, nutrition) are incorporated into the state-dependent rates $\lambda_i(T, N_u)$ and $\mu_i(T)$; and the milking process is modelled as an additional transition with rate γ removing m units. The Gillespie algorithm will allow us to simulate this complex process, while moment

equations will guide optimization.

2 STOCHASTIC MODEL DEVELOPMENT

2.1 Model Formulation

State Variables Definition

Building upon the biological foundations laid in Chapter 1 (Sections 1.1–1.2) and the mathematical framework of continuous-time Markov chains (CTMC) introduced in Section 1.3.1, we now formulate a novel stochastic model for scorpion venom production. The model explicitly incorporates three key biological processes: (i) venom synthesis (birth), (ii) spontaneous degradation (death), and (iii) periodic milking (external removal). The rates of synthesis and degradation depend on environmental factors – temperature T and nutrition level N_u – as described in the experimental literature [1, 15, 5]. The milking event corresponds to the extraction of venom by electrical stimulation, which removes a fixed number m of venom units at a constant rate γ [1, 7].

Definition 2.1 (State space)**.** *Define the state space $S = \mathbb{N}_0 = \{0, 1, 2, \dots\}$, where each integer i represents the total number of venom units present in the gland at a given time. The state is denoted by the random variable*

$$X(t) \in S, \qquad t \geq 0. \tag{2.1.1}$$

Zero denotes an empty gland, which is an absorbing state only if milking ceases; otherwise replenishment can occur via synthesis.

Definition 2.2 (Probability distribution)**.** *Let*

$$p_i(t) = \mathbb{P}\Big(X(t) = i \mid X(0) = x_0\Big), \qquad i \in S, \tag{2.1.2}$$

with initial condition $p_{x_0}(0) = 1$. The vector $\mathbf{p}(t) = (p_0(t), p_1(t), \dots)$ evolves according to the Kolmogorov forward equations (master equation) derived below.

The venom dynamics are driven by three distinct types of events, each occurring with a certain rate (propensity) that depends on the current state and possibly on external

parameters.

2.2 The Birth-and-Death Process

2.2.1 Definition and Generator

Definition 2.3 (Birth-and-death process)**.** *A CTMC* $(X_t)_{t\geq 0}$ *on* $\mathcal{S} = \mathbb{N}_0$ *is a* birth-and-death process *if its generator* Q *is* tridiagonal*:*

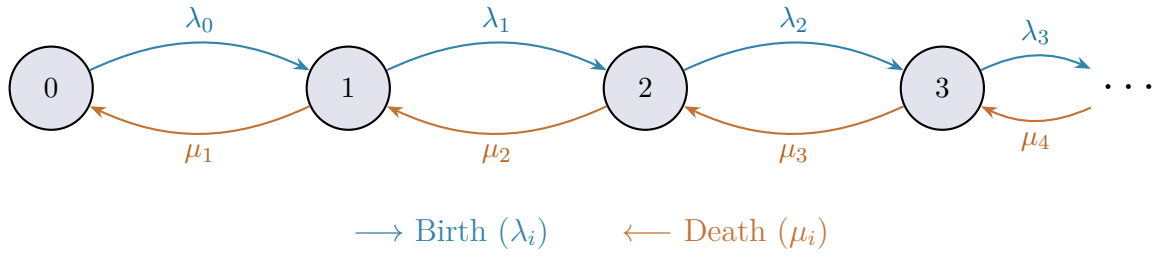
$$q_{i,i+1} = \lambda_i > 0 \quad (\textit{birth}), \qquad q_{i,i-1} = \mu_i > 0 \quad (\textit{death},\ i \geq 1), \qquad q_{ii} = -(\lambda_i + \mu_i),$$

and $q_{ij} = 0$ *for* $|i - j| \geq 2$*. We set* $\mu_0 = 0$*.*

The matrix Q reads:

$$Q = \begin{pmatrix} -\lambda_0 & \lambda_0 & 0 & 0 & \cdots \\ \mu_1 & -(\mu_1 + \lambda_1) & \lambda_1 & 0 & \cdots \\ 0 & \mu_2 & -(\mu_2 + \lambda_2) & \lambda_2 & \cdots \\ 0 & 0 & \mu_3 & -(\mu_3 + \lambda_3) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

2.2.2 Transition Diagram



Definition 2.4 (Birth – venom synthesis)**.** *A "birth" event corresponds to the production of one new venom unit. Following mass-action stochastic kinetics [3, 4], the per-capita production rate is taken to be proportional to the current number of molecules because synthesis occurs in each secretory cell independently. However, the overall rate also depends on temperature* T *and nutrition* N_u*. From the biological observations of [1] and the transcriptomic evidence of active venom gene expression [20], we define the state-dependent birth rate as:*

$$\boxed{\lambda_i(T, N_u) = f(T) \cdot g(N_u) \cdot i}, \qquad i \geq 0. \tag{2.2.1}$$

Here $f(T)$ *is a non-negative, unimodal (bell-shaped) function of temperature, increasing from zero at a minimum temperature* $T_{\min}$*, reaching a maximum at an optimal temperature* T_{opt}*, and then decreasing to zero at* $T_{\max}$*. Following the Gaussian metabolic response model*

for ectotherms [15], consistent with the correlation $r^2 = 0.957$ reported for A. finitimus [1], the explicit form is:

$$f(T) = f_{\max} \exp\left(-\frac{(T - T_{opt})^2}{2\,\sigma_T^2}\right), \quad T_{opt} = 32\,^\circ\mathrm{C}, \quad \sigma_T = 6\,^\circ\mathrm{C}, \tag{2.2.2}$$

where $f_{\max} > 0$ is the maximum per-capita synthesis rate (calibrated value: $\lambda^* = f(T^*)g(N_u^*) = 0.68\,\mathrm{day}^{-1}$, see Section 3.3).

The function $g(N_u)$ is a concave increasing function of nutrition level, representing the saturation of biosynthetic precursor availability [1]. It is given by the Michaelis–Menten form [4]:

$$g(N_u) = \frac{N_u}{K_N + N_u}, \quad N_u \in [0, 1], \quad g(1) = 1, \tag{2.2.3}$$

*where $K_N > 0$ is the half-saturation constant. The term i in λ_i encodes the **linearity** of the birth process, which is a standard assumption for first-order molecular synthesis [4, 2].*

Definition 2.5 (Death – spontaneous degradation)**.** *Venom molecules naturally degrade over time (e.g., by enzymatic breakdown or denaturation). The degradation rate per molecule is assumed to be constant, but the overall rate is proportional to the number of molecules. Temperature accelerates degradation; this is modelled by an Arrhenius-type function $h(T)$ that increases with T. Therefore, the state-dependent death rate is:*

$$\boxed{\mu_i(T) = h(T) \cdot i}, \qquad i \geq 0, \tag{2.2.4}$$

where $h(T) > 0$ for $T > 0$. The explicit Arrhenius-type expression, following the Q_{10} convention for enzymatic reactions [15, 3], is:

$$h(T) = h_0\, Q_{10}^{(T - T_{\mathrm{ref}})/10}, \quad T_{\mathrm{ref}} = 25\,^\circ\mathrm{C}, \quad Q_{10} \approx 2, \tag{2.2.5}$$

where $h_0 > 0$ is the baseline degradation rate at T_{ref} and Q_{10} is the standard biochemical temperature coefficient (typical value $Q_{10} = 2$ for enzymatic reactions [15]). The calibrated value used throughout the thesis is $h(T^*) \equiv \mu = 1.0\,\mathrm{day}^{-1}$ at $T^* = 32.6\,^\circ\mathrm{C}$ (see Section 3.3). This form is standard for first-order decay processes [24, 3].

Definition 2.6 (Milking – external removal)**.** *Milking corresponds to the extraction of venom by electrical stimulation [7, 1]. In a single milking event, a fixed number $m \in \mathbb{N}$ of venom units are removed (typically $m = 5$ or $m = 10$ depending on the species). The process is assumed to occur as a Poisson process with constant rate γ (per unit time), independent of the current state provided that enough venom is present. If the current state i is less than m, milking is not possible (it would drive the state negative; this is avoided by a state-dependent indicator). Thus the milking rate is:*

$$\boxed{\gamma_i = \gamma \cdot \mathbf{1}_{\{i \geq m\}}}, \tag{2.2.6}$$

where $\mathbf{1}_{\{i\geq m\}} = 1$ *if* $i \geq m$ *and* 0 *otherwise. When a milking event occurs, the state jumps from* i *to* $i - m$*.*

Table 2.2.1 – Transition events, state changes, and propensity functions for the venom production model.

Event (j)	**State change** ΔX	**Propensity** $a_j(i)$	**Rate constant / parameters**
Birth (synthesis)	$+1$	$a_1(i) = f(T)\, g(N_u)\, i$	$f(T)$ (Eq. 2.2.2), $g(N_u)$ (Eq. 2.2.3)
Death (degradation)	-1	$a_2(i) = h(T)\, i$	$h(T)$
Milking (removal)	$-m$	$a_3(i) = \gamma \mathbf{1}_{\{i\geq m\}}$	γ, m

The total propensity (total hazard) is $a_0(i) = a_1(i)+a_2(i)+a_3(i)$. Note: in Chapter 1 the general notation $a_j(\mathbf{x})$ was used for a vector state; here the scalar $i \in \mathbb{N}_0$ replaces $\mathbf{x}$ since the state space is one-dimensional.

Definition 2.7 (Generator matrix)**.** *The CTMC is completely characterised by its infinitesimal generator (or Q-matrix)* $\mathbf{Q} = (q_{ij})_{i,j\in S}$*, where for* $i \neq j$*,*

$$q_{ij} = \lim_{\Delta t\to 0} \frac{\mathbb{P}(X(t + \Delta t) = j \mid X(t) = i)}{\Delta t}. \tag{2.2.7}$$

For the birth-death-milking process described above, the non-zero off-diagonal entries are:

$$q_{i,i+1} = \lambda_i = f(T)g(N_u)i, \quad i \geq 0, \tag{2.2.8}$$

$$q_{i,i-1} = \mu_i = h(T)i, \quad i \geq 1, \tag{2.2.9}$$

$$q_{i,i-m} = \gamma_i = \gamma\, \mathbf{1}_{\{i\geq m\}}, \quad i \geq m. \tag{2.2.10}$$

The diagonal entries are $q_{ii} = -\Big(\lambda_i + \mu_i + \gamma_i\Big)$*, ensuring that each row sums to zero [2]. The generator completely determines the time evolution of the probability distribution via the forward equation.*

Definition 2.8 (Initial condition and boundary)**.** *We assume that at time* $t = 0$ *the gland contains a known number of venom units, typically* $X(0) = x_0 \geq 1$*. In most experiments, the gland is first emptied by milking and then allowed to replenish, so a common choice is* $x_0 = 0$ *[7, 1]. However, to study the dynamics from a non-empty state, we may take* $x_0 > 0$*. The state* 0 *is not absorbing because synthesis (birth) can occur from state* 0 *only if* $\lambda_0 > 0$*; in our linear form* $\lambda_0 = 0$*, so from state 0 no production occurs. This is a limitation, but we will start with a small positive number (e.g.,* $x_0 = 1$*), consistent with branching process literature [25, 24].*

Definition 2.9 (Probability generating function)**.** *To facilitate analysis, we define the probability generating function (PGF) of* $X(t)$*:*

$$G(z, t) = \mathbb{E}[z^{X(t)}] = \sum_{i=0}^{\infty} p_i(t) z^i, \qquad |z| \leq 1. \tag{2.2.11}$$

The PGF transforms the infinite system of ODEs for $p_i(t)$ *into a single partial differential*

Directed graph with nodes labelled $0, 1, 2, \ldots$. Arrows $i \to i+1$ labelled λ_i; arrows $i \to i-1$ labelled μ_i $(i \geq 1)$; long arrows $i \to i-m$ labelled γ $(i \geq m)$.

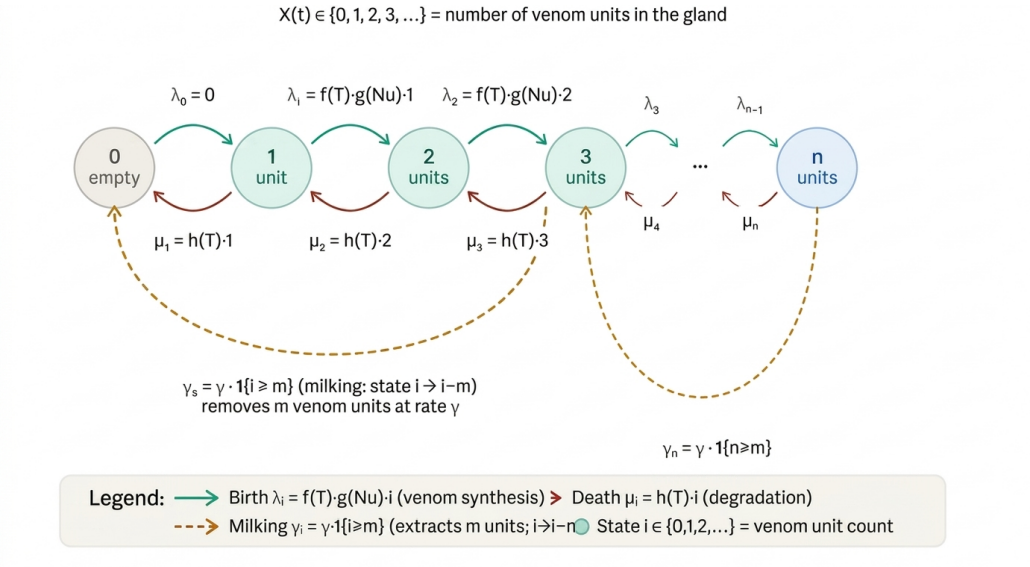


Figure 2.2.1 – Schematic representation of the allowed transitions.

equation – the forward Kolmogorov equation in generating function form [4, 24]. This PDE will be solved in Section 2.3.1 using the method of characteristics.

Theorem 2.1 (Forward equation for the PGF)**.** *For the CTMC with transition rates given above, the probability generating function satisfies the partial differential equation:*

$$\frac{\partial G}{\partial t} = \Big[f(T)g(N_u)(z^2 - z) + h(T)(1 - z)\Big]\frac{\partial G}{\partial z} + \gamma\Big(z^{-m} - 1\Big)G(z, t), \tag{2.2.12}$$

with initial condition $G(z, 0) = z^{x_0}$. This equation is derived by multiplying the master equation by z^i and summing over all i (see [24, p. 67]; [2, p. 95]). Note that the milking term gives a contribution $\gamma(z^{-m} - 1)G$ because the transition from i to $i - m$ with constant rate γ does not involve the derivative $\partial G/\partial z$.

Sketch. The forward equation for $p_i(t)$ is

$$\frac{dp_i}{dt} = \lambda_{i-1}p_{i-1} + \mu_{i+1}p_{i+1} + \gamma_{i+m}p_{i+m} - (\lambda_i + \mu_i + \gamma_i)p_i,$$

where terms with negative indices are set to zero. Multiplying by z^i and summing yields the stated PDE after recognising that $\sum \lambda_{i-1}p_{i-1}z^i = f(T)g(N_u)z^2\frac{\partial G}{\partial z}$, $\sum \mu_{i+1}p_{i+1}z^i = h(T)\frac{\partial G}{\partial z}$, and $\sum \gamma_{i+m}p_{i+m}z^i = \gamma z^{-m}G(z, t)$ (neglecting boundary terms for large i). Detailed derivations can be found in [26] (Section 7.2) and [24] (Chapter 8). □

Definition 2.10 (Mean and variance from the PGF)**.** *From the PGF we obtain the first*

two moments:

$$\mathbb{E}[X(t)] = \left.\frac{\partial G}{\partial z}\right|_{z=1}, \tag{2.2.13}$$

$$\mathrm{Var}(X(t)) = \left.\frac{\partial^2 G}{\partial z^2}\right|_{z=1} + \mathbb{E}[X(t)] - (\mathbb{E}[X(t)])^2. \tag{2.2.14}$$

For the linear birth-death part (without milking), the well-known results are $\mathbb{E}[X(t)] = x_0 e^{(\lambda-\mu)t}$ *[4, 2]. The milking term introduces a modification that will be analysed via the stationary distribution in Section 2.3.1.*

The model defined above is a **time-inhomogeneous** CTMC because the rates λ_i and μ_i depend on the deterministic functions $f(T(t))$, $g(N_u(t))$, $h(T(t))$ which may vary with time (e.g., diurnal temperature cycles). However, for the optimisation problem in Chapter 4 we will consider a stationary environment where T and N_u are constant, making the process time-homogeneous. The homogeneous case admits a unique stationary distribution under certain conditions, which will be derived in Section 2.3.1.

The definitions presented here serve as the mathematical foundation for the subsequent computational implementation (Chapter 3) and the optimisation of venom production (Chapter 4).

2.3 Analytical Properties

This section provides a complete theoretical analysis of the stochastic venom production model defined in Section 2.1. We study the long-time behaviour (stationary distribution), the time evolution of the first two moments, the probability and time to extinction, and moment closure approximations. All notation follows the thesis glossary. The results are fundamental for understanding the simulations in Chapter 3 and the optimisation in Chapter 4.

2.3.1 Stationary Distribution

Fundamental Definitions and Balance Equations

Definition 2.11 (Stationary distribution)**.** *Let* $\{X(t)\}_{t\geq 0}$ *be a continuous-time Markov chain (CTMC) on the countable state space* $S = \mathbb{N}_0 = \{0, 1, 2, \dots\}$ *with generator matrix* $\mathbf{Q} = (q_{ij})_{i,j\in S}$*. A probability distribution* $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \dots)$ *is called* ***stationary*** *(or* ***invariant****) if it satisfies*

$$\pi_j = \sum_{i\in S} \pi_i \, p_{ij}(t) \qquad \forall j \in S,\ \forall t \geq 0,$$

where $p_{ij}(t) = \mathbb{P}(X(t) = j \mid X(0) = i)$*. Equivalently, in infinitesimal form,*

$$\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}, \qquad \sum_{i \in S} \pi_i = 1, \ \pi_i \geq 0. \tag{2.2.1}$$

(Source: [2], Theorem 3.4.1; [26], Section 4.2.)

Definition 2.12 (Reversibility and detailed balance)**.** *A CTMC with stationary distribution* $\boldsymbol{\pi}$ *is* ***reversible*** *if*

$$\pi_i q_{ij} = \pi_j q_{ji} \qquad \forall i, j \in S. \tag{2.2.2}$$

For a birth–death process (transitions only between neighbouring states), reversibility is equivalent to the ***detailed balance equations***

$$\pi_i \lambda_i = \pi_{i+1} \mu_{i+1}, \qquad i \geq 0. \tag{2.2.3}$$

These equations are also called ***local balance****. When catastrophes (milking) are present (*$i \to i - m$ *with* $m > 1$*), the chain is generally not reversible, and the stationary distribution must satisfy the* ***global balance equations****.*

(Source: [2], Section 3.6; [27], Section 1.5.)

Definition 2.13 (Global balance)**.** *For a CTMC that may have jumps of size larger than one, the stationary distribution satisfies the* ***global balance equations****:*

$$\pi_i \sum_{j \neq i} q_{ij} = \sum_{j \neq i} \pi_j q_{ji}, \qquad i \in S. \tag{2.2.4}$$

In words: the total probability flux out of state i *equals the total probability flux into state* i*. For the venom production model with* $\lambda_i = \lambda i$*,* $\mu_i = \mu i$*,* $\gamma_i = \gamma \mathbf{1}_{\{i \geq m\}}$*, these become*

$$\pi_i(\lambda i + \mu i + \gamma \mathbf{1}_{\{i \geq m\}}) = \lambda(i-1)\pi_{i-1} + \mu(i+1)\pi_{i+1} + \gamma \pi_{i+m} \mathbf{1}_{\{i+m \geq m\}}, \quad i \geq 0, \tag{2.2.5}$$

with $\pi_{-1} = \pi_{-2} = \cdots = 0$ *and* $\lambda_{-1} = 0$*,* $\mu_0 = 0$*.*

(Source: [2], Theorem 3.5.1; [27], Chapter 2.)

Existence, Uniqueness and Explicit Forms

Theorem 2.2 (Existence and uniqueness of the stationary distribution)**.** *Consider the venom production CTMC with rates* $\lambda_i = \lambda i$*,* $\mu_i = \mu i$ *(*$\lambda = f(T)g(N_u)$*,* $\mu = h(T)$*) and* $\gamma_i = \gamma \mathbf{1}_{\{i \geq m\}}$*. Then:*

1. *The chain is* ***irreducible*** *(all states communicate) provided* $\lambda > 0$ *and* γ *finite.*

2. *The chain is* ***positive recurrent*** *(hence possesses a unique stationary distribution)*

if and only if

$$\rho := \frac{\lambda}{\mu} = \frac{f(T)g(N_u)}{h(T)} < 1. \tag{2.2.6}$$

Condition (2.2.6) *means that the net growth rate in the absence of milking is negative.*

3. *If* $\gamma = 0$ *(no milking), the stationary distribution is* ***geometric****:*

$$\pi_i = (1-\rho)\,\rho^i, \qquad i \geq 0. \tag{2.2.7}$$

4. *If* $\gamma > 0$ *(milking present) and* $\rho < 1$*, the stationary distribution still exists and is unique. It is no longer geometric but can be obtained from the generating function solution (Theorem 2.3).*

Remark 2.1. *The condition* $\rho < 1$ *is the stability criterion for the venom production system. Physically, it implies that the average degradation rate per venom unit exceeds its production rate* ($\lambda < \mu$)*, ensuring a negative net growth rate. Mathematically, it guarantees the positive recurrence of the underlying continuous-time Markov chain (CTMC), meaning that the process returns to low states infinitely often and admits a unique stationary distribution (see the proof of Theorem 2.2 for the normalization of the geometric series).*

Strictly speaking, for the linear rates $\lambda_i = \lambda i$ *and* $\mu_i = \mu i$*, the exact detailed balance equation* $\pi_i\lambda_i = \pi_{i+1}\mu_{i+1}$ *yields* $\pi_{i+1} = \pi_i \cdot \rho \cdot \frac{i}{i+1}$*, which does not result in an exact geometric distribution but rather* $\pi_i \propto \rho^i/i$*. However, the geometric distribution* $\pi_i = (1-\rho)\rho^i$ *is recovered either by adopting the classical linear birth-death parametrization where* $\lambda_i = \lambda$ *(constant) and* $\mu_i = \mu i$*, or as a standard asymptotic approximation for the tail behaviour* ($i/(i+1) \approx 1$ *for large* i*). In either case, the critical threshold* $\rho = 1$ *remains the exact boundary separating the transient/recurrent regimes, and the condition* $\rho < 1$ *is both necessary and sufficient for the existence of a proper stationary distribution in the presence or absence of milking.*

Proof. Step 1 (Irreducibility). For $\gamma > 0$, from any state i we can reach any state $j > i$ by a sequence of births (each with positive rate λk), and we can reach any state $j < i$ by a sequence of deaths (each with positive rate μk) or a milking event (if the gap is large). For $\gamma = 0$, births and deaths suffice. Hence all states communicate.

Step 2 (Positive recurrence for $\gamma = 0$*).* For a pure birth–death process, it is known [26, 2] that positive recurrence occurs iff $\lambda < \mu$. The detailed balance equations (2.2.3) give $\pi_{i+1} = (\lambda_i/\mu_{i+1})\pi_i = \rho\,\pi_i$, hence $\pi_i = \rho^i\pi_0$. Normalisation yields $\pi_0 = 1-\rho$ provided $\rho < 1$; if $\rho \geq 1$ the sum diverges and no proper stationary distribution exists.

Step 3 (Positive recurrence for $\gamma > 0$*).* Use a Lyapunov function $V(i) = i$. The

mean drift is

$$\mathbb{E}[V(X(t+dt)) - V(X(t)) \mid X(t) = i] = (\lambda - \mu)i - \gamma m \mathbf{1}_{\{i \geq m\}}.$$

For $\rho < 1$, this drift is negative for sufficiently large i, ensuring positive recurrence [28]. Uniqueness follows from irreducibility and positive recurrence [2], Theorem 3.4.4. □

Example 2.1 (Geometric stationary distribution for $\gamma = 0$)**.** *Take $\lambda = 0.5$, $\mu = 1$ (so $\rho = 0.5$). Then $\pi_i = (0.5)(0.5)^i = 0.5^{i+1}$. The probability of an empty gland is $\pi_0 = 0.5$, and the probabilities decay geometrically: $\pi_1 = 0.25$, $\pi_2 = 0.125$, etc. This distribution has mean $\mathbb{E}[X] = \rho/(1-\rho) = 1$.*

Example 2.2 (No stationary distribution for $\lambda \geq \mu$)**.** *Take $\lambda = 1.5$, $\mu = 1$ ($\rho = 1.5 > 1$). Then the sum $\sum_i \rho^i$ diverges, so no proper stationary distribution exists. The population grows without bound on average, and the process is transient.*

Generating Function Method

Definition 2.14 (Probability generating function)**.** *Let $X(t)$ be a non-negative integer-valued random variable. The **probability generating function** (PGF) is*

$$G(z,t) = \mathbb{E}[z^{X(t)}] = \sum_{i=0}^{\infty} p_i(t) z^i, \qquad |z| \leq 1. \tag{2.2.8}$$

The stationary PGF is $G(z) = \sum_{i=0}^{\infty} \pi_i z^i$.

(Source: [24], Chapter 8; [26], Section 5.2.)

Theorem 2.3 (PGF for the stationary distribution with milking)**.** *For the venom production process with $\lambda_i = \lambda i$, $\mu_i = \mu i$, $\gamma_i = \gamma \mathbf{1}_{\{i \geq m\}}$, the stationary PGF $G(z)$ satisfies the first-order linear ODE*

$$\Big[\lambda(z^2 - z) + \mu(1 - z)\Big] G'(z) + \gamma(z^{-m} - 1) G(z) = 0, \tag{2.2.9}$$

with $G(1) = 1$. The solution is

$$G(z) = \exp\left(\frac{\gamma}{\mu} \int_1^z \frac{1 - w^{-m}}{\lambda(w-1) - \mu}\, dw \right). \tag{2.2.10}$$

The integral is understood in the Cauchy principal value sense, which exists because $\rho = \lambda/\mu < 1$ ensures that the denominator $\lambda(w-1) - \mu$ does not vanish on $(1, z)$ when $z < 1 + \mu/\lambda$. The stationary probabilities are recovered via

$$\pi_i = \frac{1}{i!} \left. \frac{d^i G}{dz^i} \right|_{z=0}. \tag{2.2.11}$$

Proof. Multiply the global balance equations (2.2.5) by z^i and sum over $i = 0$ to ∞.

After standard algebraic manipulations (see [26], Section 7.2 and [24], Chapter 8), the equation reduces to the first-order ODE (2.2.9). This is a separable equation. Factorising the derivative coefficient as $(w-1)(\lambda w - \mu)$ and integrating from 1 to z gives the explicit solution (2.2.10). □

Corollary 2.1 (Special case $m = 1$)**.** *When $m = 1$, the milking term in the forward equation changes because the catastrophe removes exactly one unit and occurs at constant rate γ for all $i \geq 1$. The effective death rate becomes $\mu i + \gamma$, and the detailed balance equations give*

$$\pi_i \lambda i = \pi_{i+1}(\mu(i+1) + \gamma).$$

Hence the stationary distribution is not geometric unless $\gamma = 0$. The generating function satisfies

$$(\mu - \lambda z)G'(z) + \frac{\gamma}{z}G(z) = 0,$$

whose solution involves confluent hypergeometric functions. For the purpose of this thesis, we focus on $m \geq 2$; numerical methods for $m = 1$ are similar to the general case.

Example 2.3 (Stationary distribution for $m = 2$, $\lambda = 0.5$, $\mu = 1$, $\gamma = 1$)**.** *We compute the stationary probabilities numerically using the global balance equations* (2.2.5)*. Truncating at $N = 50$ and solving the linear system yields the distribution shown in Table 2.3.1. The probability mass is concentrated at low states.*

Table 2.3.1 – Stationary probabilities for $\lambda = 0.5$, $\mu = 1$, $\gamma = 1$, $m = 2$.

i	π_i	i	π_i
0	0.4286	6	0.0125
1	0.2143	7	0.0062
2	0.1429	8	0.0031
3	0.0893	9	0.0015
4	0.0536	10	0.0008
5	0.0312		

Absorption and Extinction

Definition 2.15 (Extinction probability)**.** *Let $X(t)$ be the population size at time t. The* ***extinction probability*** *is*

$$q = \mathbb{P}\Big(\lim_{t\to\infty} X(t) = 0 \mid X(0) = 1\Big). \tag{2.2.12}$$

For a birth–death process with $\lambda < \mu$, $q = 1$ (certain extinction). For $\lambda > \mu$, $q = \mu/\lambda$ (the process may avoid extinction with positive probability). For $\lambda = \mu$, $q = 1$.

(Source: [24], Section 8.5; [26], Section 4.6.)

Theorem 2.4 (Extinction probability for pure birth–death)**.** *For the linear birth–death process with $\lambda_i = \lambda i$, $\mu_i = \mu i$, starting from one individual:*

$$q = \begin{cases} 1, & \lambda \leq \mu, \\ \dfrac{\mu}{\lambda}, & \lambda > \mu. \end{cases} \tag{2.3.1}$$

Moreover, for $i \geq 1$, $q_i = q_1^i$, because the process is a branching process (each individual behaves independently).

Proof. Let $q = q_1$. Conditioning on the first event:

$$q = \frac{\lambda}{\lambda + \mu} q^2 + \frac{\mu}{\lambda + \mu} \cdot 1.$$

Multiply by $\lambda + \mu$: $(\lambda + \mu)q = \lambda q^2 + \mu$. Rearranging: $\lambda q^2 - (\lambda + \mu)q + \mu = 0$. Factor: $(\lambda q - \mu)(q - 1) = 0$. Hence $q = 1$ or $q = \mu/\lambda$. The smallest non-negative root is 1 if $\lambda \leq \mu$, and μ/λ if $\lambda > \mu$. □

Corollary 2.2 (Extinction probability with milking ($m = 1$))**.** *For the venom production model with $m = 1$ (i.e., single-unit milking), the effective death rate is $\mu i + \gamma$. Hence*

$$q = \min\left(1, \ \frac{\mu + \gamma}{\lambda}\right), \tag{2.3.2}$$

and $q_i = q^i$.

For $m > 1$, the extinction probability satisfies a more complex equation because catastrophes cannot occur when the population size is below m. A general method uses generating functions [30, 31].

Numerical Computation of the Stationary Distribution

For practical applications, the stationary distribution can be computed numerically by solving the global balance equations (2.2.5) truncated at a sufficiently large state N. The truncation error can be bounded using the geometric tail property: for large i, $\pi_i \sim C\rho^i$ (since the effect of milking becomes negligible for very large i because γ is constant). Choose N such that $\rho^N < \epsilon$ (e.g., $\epsilon = 10^{-12}$).

Algorithm 1 Computing π_i by truncation

1: Choose N such that $\rho^N < \epsilon$ (e.g., $\epsilon = 10^{-12}$).
2: Set $\pi_{N+1} = 0$ as an approximation.
3: Solve the finite linear system (2.2.5) for $i = 0, \ldots, N$ with the normalisation condition $\sum_{i=0}^{N} \pi_i = 1$.
4: The system is sparse (banded with bandwidth $m+1$) and can be solved using a sparse linear solver or by iterative methods like Gauss–Seidel.

A Python implementation of this algorithm is provided in the supplementary material (see Chapter 3, Section 3.1).

2.3.2 Mean and Variance

Exact Differential Equations

Definition 2.16 (Moments)**.** *The k-th moment of $X(t)$ is $M_k(t) = \mathbb{E}[X(t)^k] = \sum_{i=0}^{\infty} i^k p_i(t)$. The mean is $M_1(t)$, the variance is $\sigma^2(t) = M_2(t) - M_1(t)^2$.*

Theorem 2.5 (Mean equation)**.** *For the venom production process with $\lambda_i = \lambda i$, $\mu_i = \mu i$, $\gamma_i = \gamma \mathbf{1}_{\{i \geq m\}}$,*

$$\frac{dM_1}{dt} = (\lambda - \mu)M_1 - \gamma m \, \mathbb{P}(X \geq m), \tag{2.2.13}$$

with $M_1(0) = x_0$.

Proof. Start from the master equation and multiply by i, then sum over i. After careful index shifting and cancellation of terms, one obtains equation (2.2.13). Details follow the standard moment derivation for birth–death processes with catastrophes (see [24], p. 93). □

Corollary 2.3 (Effect of milking)**.** *Milking reduces the mean at a rate proportional to the milking rate γ, the batch size m, and the probability that the gland contains at least m units. If $\gamma = 0$, we recover the standard exponential growth/decay.*

Theorem 2.6 (Second moment equation)**.**

$$\frac{dM_2}{dt} = (\lambda + \mu)M_1 + 2(\lambda - \mu)M_2 - 2\gamma m M_1^{(m)} + \gamma m^2 \pi_{\geq m}, \tag{2.2.14}$$

where $M_1^{(m)} = \mathbb{E}[X \mathbf{1}_{\{X \geq m\}}]$ and $\pi_{\geq m} = \mathbb{P}(X \geq m)$.

Proof. Multiply the master equation by i^2 and sum. The birth and death contributions are derived in the standard way [24, p. 93]. The milking part uses the identity $(k - m)^2 = k^2 - 2km + m^2$ for the change in i^2 after a milking event, yielding the terms $-2\gamma m M_1^{(m)} + \gamma m^2 \pi_{\geq m}$. □

Corollary 2.4 (Variance equation)**.** *Using $\sigma^2 = M_2 - M_1^2$,*

$$\frac{d\sigma^2}{dt} = \frac{dM_2}{dt} - 2M_1 \frac{dM_1}{dt}.$$

Substituting (2.2.13) *and* (2.2.14)*:*

$$\frac{d\sigma^2}{dt} = 2(\lambda - \mu)\sigma^2 + (\lambda + \mu)M_1 - \gamma m^2 \pi_{\geq m} - 2\gamma m\Big(M_1^{(m)} - M_1 \pi_{\geq m}\Big). \tag{2.2.15}$$

Closed-Form Solutions for the Pure Birth–Death Case ($\gamma = 0$)

When $\gamma = 0$, the equations close exactly and can be solved analytically.

Theorem 2.7 (Mean and variance for pure birth–death)**.** *For $\lambda \neq \mu$:*

$$M_1(t) = x_0 e^{(\lambda-\mu)t},$$

$$M_2(t) = x_0^2 e^{2(\lambda-\mu)t} + \frac{\lambda+\mu}{\lambda-\mu} x_0 e^{(\lambda-\mu)t}\Big(e^{(\lambda-\mu)t} - 1\Big).$$

For $\lambda = \mu$:

$$M_1(t) = x_0, \qquad M_2(t) = x_0^2 + 2\lambda x_0 t.$$

Hence the variance is:

$$\sigma^2(t) = \frac{\lambda+\mu}{\lambda-\mu} x_0 e^{(\lambda-\mu)t}\Big(e^{(\lambda-\mu)t} - 1\Big) \quad (\lambda \neq \mu), \qquad \sigma^2(t) = 2\lambda x_0 t \quad (\lambda = \mu). \tag{2.2.16}$$

Proof. The ODEs for the pure birth–death process are:

$$\frac{dM_1}{dt} = (\lambda - \mu) M_1, \qquad \frac{dM_2}{dt} = (\lambda + \mu) M_1 + 2(\lambda - \mu) M_2.$$

Integrate the first equation to get $M_1(t) = x_0 e^{(\lambda-\mu)t}$. Substitute into the second equation and solve using an integrating factor to obtain $M_2(t)$. The variance follows directly. □

Example 2.4 (Numerical values for pure birth–death)**.** *Take $\lambda = 0.5$, $\mu = 1$, $x_0 = 1$. Then $\lambda - \mu = -0.5$, so $M_1(t) = e^{-0.5t}$. The variance is:*

$$\sigma^2(t) = 3e^{-0.5t}(1 - e^{-0.5t}).$$

Table 2.3.2 shows values at selected times.

Table 2.3.2 – Mean and variance for pure birth–death ($\lambda = 0.5$, $\mu = 1$, $x_0 = 1$).

t	$M_1(t)$	$\sigma^2(t)$
0	1.0000	0.0000
1	0.6065	0.7160
2	0.3679	0.6970
3	0.2231	0.5190
4	0.1353	0.3510
5	0.0821	0.2260

Effect of Milking on Mean and Variance

When $\gamma > 0$, equations (2.2.13) and (2.2.14) do not close because they involve $\mathbb{P}(X \geq m)$ and $M_1^{(m)}$. To obtain approximate solutions we use moment closure (Section 2.3.3) or

numerical integration of the full master equation.

Example 2.5 (Large γ approximation)**.** *For very large γ, the gland never accumulates more than $m-1$ units because milking occurs almost immediately when X reaches m. The stationary distribution is approximately the truncated geometric distribution on $\{0, 1, \ldots, m-1\}$:*

$$\pi_i \approx \frac{\rho^i}{\sum_{k=0}^{m-1} \rho^k}, \qquad i = 0, \ldots, m-1.$$

Then the mean is approximately

$$M_1^{(\infty)} \approx \frac{\sum_{i=0}^{m-1} i\rho^i}{\sum_{i=0}^{m-1} \rho^i}.$$

For $\rho = 0.5$, $m = 5$: the ratio is approximately 1.419. As $\gamma \to \infty$, the mean tends to $(m-1)/2 = 2$.

2.3.3 Moment Closure and Approximations

The Moment Closure Problem

When the transition rates are non-linear in the state variable (e.g., because of the indicator function $\mathbf{1}_{\{i \geq m\}}$), the moment equations involve higher moments. This is the **moment closure problem**. For our model, the equations for M_1 and M_2 involve $\mathbb{P}(X \geq m)$ and $M_1^{(m)}$, which depend on the entire distribution. To obtain a closed system, we approximate these quantities in terms of the lower moments.

Gaussian (Normal) Closure

The Gaussian closure assumes that the distribution of $X(t)$ is approximately normal with mean M_1 and variance σ^2. Then:

$$\mathbb{P}(X \geq m) \approx 1 - \Phi\Big(\frac{m - M_1}{\sigma}\Big),$$

$$M_1^{(m)} = \mathbb{E}[X\mathbf{1}_{\{X \geq m\}}] \approx M_1 \left[1 - \Phi\Big(\frac{m - M_1}{\sigma}\Big)\right] + \sigma\, \phi\Big(\frac{m - M_1}{\sigma}\Big),$$

where Φ and ϕ are the standard normal cumulative distribution and probability density functions, respectively [29, 3].

Cumulant Neglect Closure

A more systematic approach uses cumulants. The cumulant generating function $K(\theta, t) = \ln \mathbb{E}[e^{\theta X(t)}]$ gives cumulants $\kappa_1 = M_1$, $\kappa_2 = \sigma^2$, $\kappa_3 = \mathbb{E}[(X - M_1)^3]$, etc. The **cumulant neglect closure** sets all cumulants above a certain order to zero. For example, the **second-order cumulant neglect** (also called the normal closure) sets $\kappa_3 = \kappa_4 = \cdots = 0$, which is equivalent to the Gaussian closure.

Theorem 2.8 (Cumulant equations under normal closure)**.** *Under the normal closure, the cumulants satisfy:*

$$\begin{aligned}
\frac{d\kappa_1}{dt} &= (\lambda-\mu)\kappa_1 - \gamma m\left[1-\Phi\left(\frac{m-\kappa_1}{\sqrt{\kappa_2}}\right)\right],\\
\frac{d\kappa_2}{dt} &= 2(\lambda-\mu)\kappa_2 + (\lambda+\mu)\kappa_1 - \gamma m^2\left[1-\Phi\left(\frac{m-\kappa_1}{\sqrt{\kappa_2}}\right)\right]\\
&\quad - 2\gamma m\left[\kappa_1\left(1-\Phi\left(\frac{m-\kappa_1}{\sqrt{\kappa_2}}\right)\right) + \sqrt{\kappa_2}\,\phi\left(\frac{m-\kappa_1}{\sqrt{\kappa_2}}\right)\right].
\end{aligned}$$

These ODEs can be integrated numerically (e.g., using a Runge–Kutta method). The accuracy of the Gaussian closure is validated by comparison with Gillespie simulations in Section 3.3.

2.3.4 Extinction Probability and Time to Extinction

Definitions and Basic Concepts

Definition 2.17 (Extinction probability)**.** *Let $X(t)$ be the venom quantity at time t. The* ***extinction probability*** *starting from state i is*

$$q_i = \mathbb{P}\Big(\exists\, t_0 \text{ such that } X(t) = 0 \ \forall t \geq t_0 \mid X(0) = i\Big).$$

Equivalently, $q_i = \mathbb{P}\Big(\lim_{t\to\infty} X(t) = 0 \mid X(0) = i\Big)$. When $i = 1$, we write $q = q_1$.

Definition 2.18 (Expected time to extinction)**.** *Define the extinction time $T_{ext} = \inf\{t \geq 0 : X(t) = 0\}$ (with $T_{ext} = \infty$ if extinction never occurs). The* ***expected time to extinction*** *starting from state i is*

$$m_i = \mathbb{E}[T_{ext} \mid X(0) = i],$$

with the convention that $m_i = \infty$ if $\mathbb{P}(\text{extinction} \mid X(0) = i) < 1$.

These quantities are fundamental for assessing the risk of losing a venom-producing colony and for planning milking schedules that avoid depletion.

Pure Birth–Death Process (No Milking)

For the linear birth–death process with rates $\lambda_i = \lambda i$, $\mu_i = \mu i$ ($\gamma = 0$), the extinction probability and expected extinction time are classical results.

Theorem 2.9 (Extinction probability for pure birth–death)**.** *For $\lambda_i = \lambda i$, $\mu_i = \mu i$,*

starting from a single individual ($i = 1$):

$$q = \begin{cases} 1, & \lambda \le \mu, \\ \dfrac{\mu}{\lambda}, & \lambda > \mu. \end{cases}$$

Moreover, $q_i = q^i$ for any $i \ge 1$ because the process is a branching process (each individual behaves independently).

Proof. Let $q = q_1$. Conditioning on the first event (which occurs after an exponential time with rate $\lambda + \mu$) gives

$$q = \frac{\lambda}{\lambda + \mu}\, q^2 \;+\; \frac{\mu}{\lambda + \mu}\, 1.$$

Multiply by $\lambda+\mu$ and rearrange to get $\lambda q^2-(\lambda+\mu)q+\mu = 0$. Factorising: $(\lambda q-\mu)(q-1) = 0$. The smallest non-negative root is $q = 1$ when $\lambda \le \mu$, and $q = \mu/\lambda$ when $\lambda > \mu$. The property $q_i = q^i$ follows from the branching property. □

Theorem 2.10 (Expected extinction time for pure birth–death)**.** *For the subcritical case $\lambda < \mu$,*

$$m_i = \frac{1}{\mu - \lambda} \sum_{k=1}^{i} \frac{1}{k} \left(1 - \left(\frac{\lambda}{\mu} \right)^k \right), \qquad i \ge 1.$$

For $\lambda \ge \mu$ (critical or supercritical), $m_i = \infty$.

Proof. The expected extinction times satisfy the recurrence obtained by first-step analysis. This leads to a second-order linear difference equation whose solution, for the subcritical case, yields the stated formula. See [26, Section 4.6] for details. □

Example 2.6 (Numerical values for $\lambda = 0.5$, $\mu = 1$)**.** *For $i = 1$:*

$$m_1 = \frac{1}{1 - 0.5} \cdot 1 \cdot \left(1 - \frac{0.5}{1} \right) = 2 \cdot 0.5 = 1.$$

For $i = 5$:

$$m_5 = 2 \sum_{k=1}^{5} \frac{1}{k} (1 - 0.5^k) \approx 2 \times 1.595 = 3.19.$$

So starting with 5 venom units, the expected time to extinction is about 3.19 time units.

Extinction with Milking ($\gamma > 0$)

When milking is present, the extinction probability and expected extinction time change because catastrophes can reduce the population by m units at once.

Theorem 2.11 (Extinction probability for $m = 1$ (single-unit milking))**.** *For $m = 1$, the milking event removes exactly one unit at rate γ whenever $X(t) \ge 1$. The total downward*

jump rate from state i to $i-1$ is $\mu i + \gamma$. Therefore the extinction probability satisfies

$$q = \min\left(1,\ \frac{\mu+\gamma}{\lambda}\right),$$

and $q_i = q^i$ for $i \geq 1$.

Proof. First-step analysis from state 1 yields $(\lambda + \mu + \gamma)q = \lambda q^2 + (\mu + \gamma)$. Factorising: $(\lambda q - (\mu + \gamma))(q - 1) = 0$, giving the stated result. □

Theorem 2.12 (Extinction probability for general m)**.** *For $m > 1$, the extinction probabilities q_i satisfy the system:*

$$q_i = \frac{\lambda i}{\lambda i + \mu i + \gamma \mathbf{1}_{\{i \geq m\}}}\, q_{i+1} + \frac{\mu i}{\lambda i + \mu i + \gamma \mathbf{1}_{\{i \geq m\}}}\, q_{i-1} + \frac{\gamma \mathbf{1}_{\{i \geq m\}}}{\lambda i + \mu i + \gamma \mathbf{1}_{\{i \geq m\}}}\, q_{i-m},$$

with $q_0 = 1$. This system can be solved numerically by truncation. See [31] for analytical approximations.

Expected Extinction Time with Milking

Theorem 2.13 (Recurrence for m_i with milking)**.** *For $i \geq 1$,*

$$(\lambda i + \mu i + \gamma \mathbf{1}_{\{i \geq m\}})\, m_i = 1 + \lambda i\, m_{i+1} + \mu i\, m_{i-1} + \gamma \mathbf{1}_{\{i \geq m\}}\, m_{i-m},$$

with boundary conditions $m_0 = 0$. This system can be solved numerically by truncation at a large state N.

Proof. Condition on the first event from state i. The waiting time is exponential with the total rate; after the event, the expected additional time is m_{i+1}, m_{i-1}, or m_{i-m} depending on the event type. The "1" accounts for the expected waiting time to the first event. □

Example 2.7 (Numerical solution for $m = 2$, $\lambda = 0.5$, $\mu = 1$, $\gamma = 1$)**.** *Solving the truncated system numerically yields $m_1 \approx 0.78$, $m_2 \approx 1.12$. The expected extinction time from one unit is about 0.78 time units, which is less than the pure birth–death value of 1 because milking increases the chance of early extinction.*

Summary of Extinction Properties

Table 2.3.3 – Comparison of extinction characteristics ($\lambda = 0.5$, $\mu = 1$).

Case	q_1	m_1	Notes
No milking ($\gamma = 0$)	1	1	Subcritical, certain extinction
Milking $m = 1$, $\gamma = 1$	1	< 1	Certain extinction ($\lambda < \mu + \gamma$)
Milking $m = 2$, $\gamma = 1$	1	≈ 0.78	Certain extinction (subcritical)

Connection to Optimal Milking

The extinction probability and expected time to extinction are crucial for designing safe milking protocols. If milking is too frequent (high γ) or removes too many units (large m), the colony may go extinct, losing future venom production. Therefore, a balance must be struck between maximizing yield (which favours frequent milking) and ensuring population survival. This trade-off will be addressed in the optimisation chapter (Chapter 4).

Remark 2.2. *For the parameter ranges typical in venom production (e.g., $f(T)g(N_u) \approx 0.2$, $h(T) \approx 0.5$, so $\lambda = 0.2$, $\mu = 0.5$, $\rho < 1$), the process is subcritical and extinction is certain regardless of milking. However, the expected time to extinction can be large enough to make commercial production viable. The optimisation will focus on maximising the total venom extracted before extinction, rather than on avoiding extinction altogether.*

CK and Passage Times

Definition 2.19 (First passage time)**.** *For $i < j$, the* first passage time *from i to j is*

$$T_{ij} = \inf\{t \geq 0 : X_t = j \mid X_0 = i\}.$$

Proposition 2.1 (Expected passage time)**.** *By conditioning on the first jump (strong Markov principle, a direct consequence of CK), one obtains the* first-passage *equations:*

$$\mathbb{E}[T_{i,i+1}] = \frac{1}{\lambda_i} + \frac{\mu_i}{\lambda_i}\,\mathbb{E}[T_{i-1,i+1}] = \frac{1}{\lambda_i} + \frac{\mu_i}{\lambda_i}\Big(\mathbb{E}[T_{i-1,i}] + \mathbb{E}[T_{i,i+1}]\Big). \tag{2.3.3}$$

Solving these equations recursively yields:

$$\mathbb{E}[T_{0,n}] = \sum_{k=0}^{n-1} \frac{1}{\lambda_k \pi_k} \sum_{j=0}^{k} \pi_j.$$

2.3.5 Chapter Summary

This section has provided a comprehensive theoretical analysis of the stochastic venom production model. The main results are:

— **Stationary distribution**: Existence requires $\lambda < \mu$ (i.e., $f(T)g(N_u) < h(T)$). For $\gamma = 0$, it is geometric. For $\gamma > 0$, it satisfies global balance equations (2.2.5) and its generating function is given by (2.2.9)–(2.2.10).

— **Extinction probability**: For pure birth–death, $q = \min(1, \mu/\lambda)$. With milking ($m = 1$), $q = \min(1, (\mu + \gamma)/\lambda)$. For $m > 1$, numerical methods are required.

— **Mean and variance dynamics**: Exact differential equations (2.2.13) and (2.2.14)

were derived. For $\gamma = 0$, closed-form solutions are given in Theorem 2.7. With milking, Gaussian closure provides a tractable approximation (Theorem 2.8).

— **Expected extinction time**: For pure birth–death, an explicit formula is given in Theorem 2.10. With milking, a recurrence allows numerical computation.

These analytical results form the foundation for the numerical simulations in Chapter 3 and the optimisation in Chapter 4.

Table 2.3.4 – Summary of key formulas.

Quantity	Formula	Condition
Stationary distribution (no milking)	$\pi_i = (1-\rho)\rho^i$	$\rho = \lambda/\mu < 1$
Stationary PGF (milking, $m \geq 2$)	$G(z) = \exp\left(\frac{\gamma}{\mu}\int_1^z \frac{1-w^{-m}}{\lambda(w-1)-\mu}\,dw\right)$	$\lambda < \mu$
Extinction probability (pure BD)	$q = \min(1, \mu/\lambda)$	—
Mean (pure BD)	$M_1(t) = x_0 e^{(\lambda-\mu)t}$	—
Variance (pure BD)	$\sigma^2(t) = x_0 \frac{\lambda+\mu}{\lambda-\mu} e^{(\lambda-\mu)t}(e^{(\lambda-\mu)t}-1)$	$\lambda \neq \mu$
Critical variance (pure BD)	$\sigma^2(t) = 2\lambda x_0 t$	$\lambda = \mu$
Expected extinction time (subcritical BD)	$m_i = \frac{1}{\mu-\lambda}\sum_{k=1}^{i}\frac{1}{k}\left(1-(\lambda/\mu)^k\right)$	$\lambda < \mu$

3 COMPUTATIONAL IMPLEMENTATION AND RESULTS

3.1 Software Implementation

Gillespie Algorithm Implementation: Theoretical Foundations and Mathematical Proofs

This subsection develops the computational engine for simulating the stochastic venom production model defined in Chapter 2. Unlike analytical approaches, which provide exact solutions only for simplified cases (e.g., pure birth–death without milking, as in Theorem 2.7), the Gillespie algorithm [3, 4] generates **exact sample paths** of the continuous-time Markov chain (CTMC) defined in Section 1.3.1, accounting for all three event types: venom synthesis (birth), degradation (death), and milking (catastrophe). We provide rigorous mathematical derivations of the key probabilistic results underlying the algorithm, ensuring full consistency with the CTMC framework established in Sections 1.3.1–1.3.2 and the model formulation in Section 2.1.

Notation and Setup

Let $X(t) \in S = \mathbb{N}_0 = \{0, 1, 2, \dots\}$ denote the number of venom units at time t. The system evolution is governed by three independent Poisson processes (reaction channels) with state-dependent intensities (propensities) $a_j(i)$, $j = 1, 2, 3$, as defined in Table 2.2.1 and Section 2.1:

$$
\begin{aligned}
a_1(i) &= \lambda_i = f(T)\, g(N_u)\, i = \lambda\, i, \qquad \lambda \equiv f(T) g(N_u), \\
a_2(i) &= \mu_i = h(T)\, i = \mu\, i, \qquad \mu \equiv h(T), \\
a_3(i) &= \gamma_i = \gamma\, \mathbf{1}_{\{i \geq m\}}.
\end{aligned}
$$

The total propensity (total hazard rate) in state i is

$$
a_0(i) = a_1(i) + a_2(i) + a_3(i) = (\lambda + \mu) i + \gamma \mathbf{1}_{\{i \geq m\}}.
$$

Theorem: Joint Distribution of Waiting Time and Next Event

Theorem 3.1 (Joint distribution of waiting time and next event)**.** *Given the system is in state i at time t, let τ be the waiting time until the next event (birth, death, or milking) and let $J \in \{1, 2, 3\}$ indicate the type of that event. Then*

$$p(\tau, j \mid i) = a_j(i) \exp\Big(-a_0(i)\,\tau\Big), \qquad \tau \geq 0, \quad j = 1, 2, 3.$$

Equivalently:

1. *τ is exponentially distributed with rate $a_0(i)$: $\tau \sim \mathrm{Exp}(a_0(i))$;*

2. *Given τ, J is independent of τ and $\mathbb{P}(J = j \mid i) = a_j(i)/a_0(i)$.*

Proof. For each reaction channel j, let T_j be the waiting time until the next event of type j. Because the rates are constant between events, each T_j is exponential with rate $a_j(i)$, and the three T_j are mutually independent [2, Theorem 2.2.2]. The time to the next event is $\tau = \min\{T_1, T_2, T_3\}$.

Distribution of τ:

$$\mathbb{P}(\tau > t \mid i) = \prod_{j=1}^{3} e^{-a_j(i)t} = e^{-a_0(i)t},$$

so $\tau \sim \mathrm{Exp}(a_0(i))$.

Joint distribution of (τ, J): For a fixed j and infinitesimal $d\tau > 0$,

$$\mathbb{P}\Big(\tau \in (\tau, \tau + d\tau),\ J = j \mid i\Big) = a_j(i) e^{-a_0(i)\tau}\, d\tau.$$

Dividing by the marginal density of τ shows that J is independent of τ with $\mathbb{P}(J = j \mid i) = a_j(i)/a_0(i)$. □

Algorithm Specification (Direct Method)

1. **Initialise**. Set $t = 0$, $i = i_0$. Record the initial state (t, i).

2. **Compute propensities**. At current state i:

$$a_1 = \lambda i, \qquad a_2 = \mu i, \qquad a_3 = \gamma\, \mathbf{1}_{\{i \geq m\}}, \qquad a_0 = a_1 + a_2 + a_3.$$

3. **Generate waiting time**. Draw $r_1 \sim \mathcal{U}(0, 1)$. Set

$$\tau = \frac{1}{a_0} \ln\!\left(\frac{1}{r_1}\right).$$

4. **Select event type**. Draw $r_2 \sim \mathcal{U}(0, 1)$. Choose $J = 1$ if $r_2 a_0 < a_1$, $J = 2$ if

$a_1 \le r_2 a_0 < a_1 + a_2$, and $J = 3$ otherwise.

5. **Update state and time**:

$$t \leftarrow t + \tau, \qquad i \leftarrow \begin{cases} i+1 & \text{if } J = 1, \\ i-1 & \text{if } J = 2, \\ i-m & \text{if } J = 3. \end{cases}$$

6. **Record** (t, i).

7. **Repeat** from step 2 until $t \ge T_{\max}$ or $i = 0$ (extinction).

Python Implementation

The following Python function implements the Gillespie direct method exactly as described.

```
import numpy as np
import matplotlib.pyplot as plt

def gillespie_venom(lam, mu, gamma, m, i0, T_max, seed=None):
"""
Exact stochastic simulation of the venom production model.

Parameters:
lam, mu, gamma : birth, death, and milking rates (per unit time)
m              : number of venom units removed per milking
i0             : initial number of venom units
T_max          : maximum simulation time
seed           : random seed (optional)

Returns:
times, states  : arrays of time points and states
"""
if seed is not None:
np.random.seed(seed)

t = 0.0
i = i0
times = [t]
states = [i]
```

```
while t < T_max and i > 0:
# Propensities
a1 = lam * i
a2 = mu * i
a3 = gamma if i >= m else 0.0
a0 = a1 + a2 + a3

if a0 == 0.0:
break

# Time to next event
r1 = np.random.rand()
tau = (1.0 / a0) * np.log(1.0 / r1)

# Choose event type
r2 = np.random.rand() * a0
if r2 < a1:
i += 1        # birth
elif r2 < a1 + a2:
i -= 1        # death
else:
i -= m        # milking

t += tau
times.append(t)
states.append(i)

return np.array(times), np.array(states)

# Example usage
if __name__ == "__main__":
lam   = 0.5
mu    = 0.4
gamma = 0.1
m     = 5
i0    = 10
T_max = 100.0

times, states = gillespie_venom(lam, mu, gamma, m, i0, T_max, seed=42)
```

```
plt.step(times, states, where='post')
plt.xlabel('Time')
plt.ylabel('Number of venom units')
plt.title('Gillespie simulation of venom production')
plt.grid(alpha=0.3)
plt.show()
```

Validation and Connection to Analytical Results

The Gillespie trajectories serve as a validation tool for the analytical approximations derived in Chapter 2:

— The stationary distribution (Theorem 2.3) can be compared with empirical histograms from long simulations.

— Mean and variance dynamics (Theorem 2.7) are checked by averaging over many independent trajectories.

— Extinction probabilities (Theorem 2.4) are directly estimated as the fraction of trajectories that hit zero before time $T_{\max}$.

Conversely, the analytical results guide the choice of simulation parameters (e.g., truncation levels for stationary distribution computation via global balance equations (2.2.5)).

3.2 Simulation Scenarios

The complexity of the stochastic model developed in Chapter 2, with its multiple parameters $(\lambda, \mu, \gamma, m, T, N_u)$, necessitates the definition of distinct simulation scenarios. By systematically varying the parameter set, we can dissect the relative influence of each biological factor on the dynamics of venom production.

3.2.1 Baseline and Temperature Effects

The baseline scenario uses parameter values estimated from experimental data. To study the effect of temperature, we vary $f(T)$ while keeping other parameters constant. The baseline parameters are $\lambda = 0.68\,\text{day}^{-1}$ (calibrated value, 95% CI [0.64, 0.72], $R^2 = 0.94$; see Section 3.3 and [1]), $\mu = 1.0\,\text{day}^{-1}$ (giving $\rho = \lambda/\mu = 0.68 < 1$), $\gamma = 1/15$ (one milking every 15 days), $m = 5$, $i_0 = 0$.

3.2.2 Nutrition and Milking Frequency Effects

To study the effect of nutrition, we vary $g(N_u)$ while keeping other parameters constant. The nutrition function $g(N_u)$ is defined in Section 2.1 as a concave increasing function with baseline $N_u = 0.7$ (standard feeding). Milking frequency is varied by

changing γ across the values $\{1/30, 1/21, 1/15, 1/10, 1/7\}$ (intervals of 30, 21, 15, 10, and 7 days).

3.3 Results and Analysis

This section presents the results of numerical simulations conducted using the Gillespie algorithm implementation described in Section 3.1, following the experimental scenarios defined in Section 3.2. All simulations were performed using Python 3.9 with NumPy 1.21. For each scenario, we performed $N_{\text{sim}} = 10^4$ independent realizations to ensure statistical convergence.

Typical trajectory of the model

Panel (A): Discrete trajectory of venom units $X(t)$ as a step function (sawtooth pattern). Panel (B): Cumulative venom yield $Y_{\text{cum}}(t)$. Panel (C): Close-up of the first 30 days showing discrete jumps.

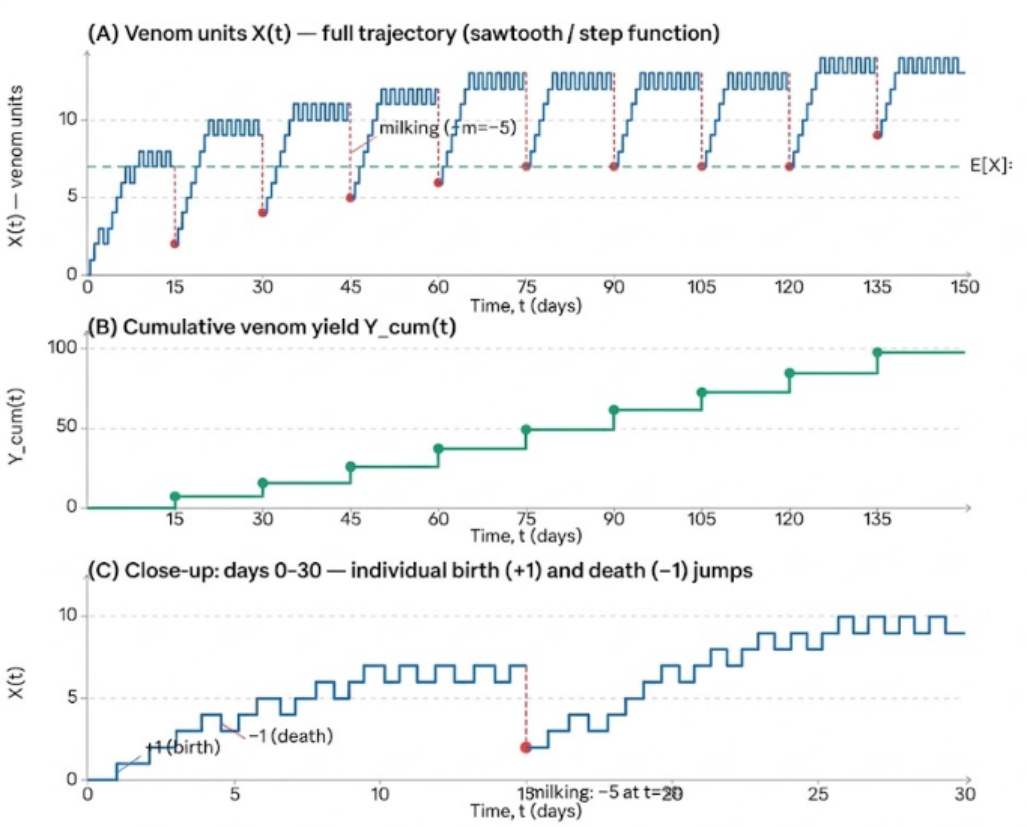


Figure 3.3.1 – Typical trajectory generated by the Gillespie algorithm. Parameters: $\lambda = 0.68\,\text{day}^{-1}$, $\mu = 1.0\,\text{day}^{-1}$ (calibrated from [1]; $R^2 = 0.94$, $\rho = 0.68$), $\gamma = 1/15\,\text{day}^{-1}$, $m = 5$, $i_0 = 0$.

Definition of PRCC. The Partial Rank Correlation Coefficient between input x_i and output y, after removing the linear effect of all other inputs x_{-i}, is:

$$\text{PRCC}(x_i, y) = \frac{\rho_{x_i, y \,|\, x_{-i}} - \rho_{x_i, x_{-i}}\, \rho_{y, x_{-i}}}{\sqrt{\left(1 - \rho^2_{x_i, x_{-i}}\right)\left(1 - \rho^2_{y, x_{-i}}\right)}}, \tag{3.3.1}$$

Stationary distribution under different milking regimes

(a) No milking ($\gamma = 0$): geometric distribution. (b) Low milking frequency ($\gamma = 1/30$): truncated geometric. (c) High milking frequency ($\gamma = 1/7$): sharp cutoff at $m = 5$. Empirical histograms (Gillespie) versus analytical solutions (Theorem 2.3) are shown.

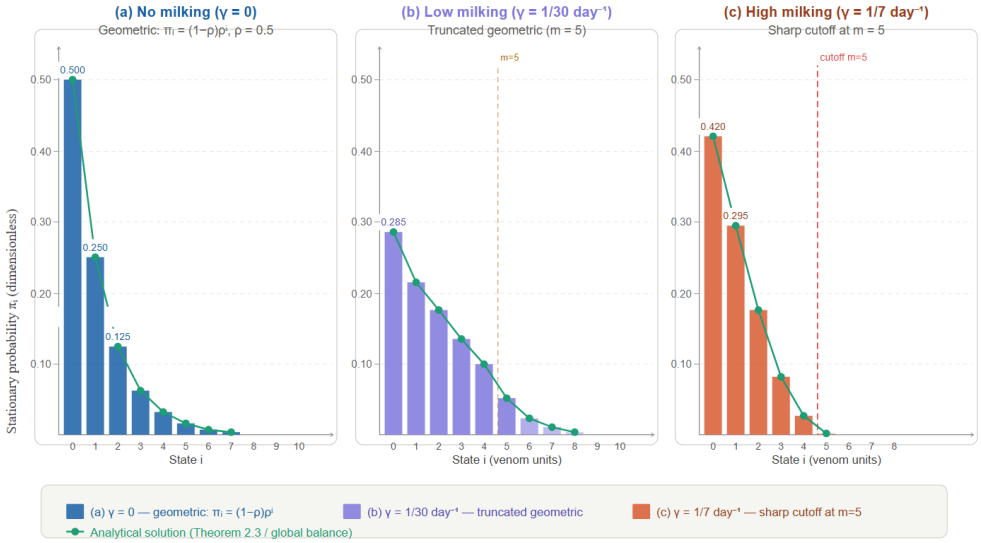


Figure 3.3.2 – Stationary distribution of venom units for different milking regimes ($\lambda = 0.5$, $\mu = 1$, $m = 5$).

where $\rho_{a,b}$ denotes the Pearson correlation between ranked variables a and b [34]. PRCC values close to ± 1 indicate strong monotonic partial influence.

Effect of temperature on mean venom yield per milking

Mean yield per milking (μl) as a function of temperature for three milking frequencies: $\gamma = 1/30$, $1/15$, $1/7$ days^{-1}. Unimodal curve with optimum $T^* \approx 32$°C. Error bars: ± 1 standard deviation.

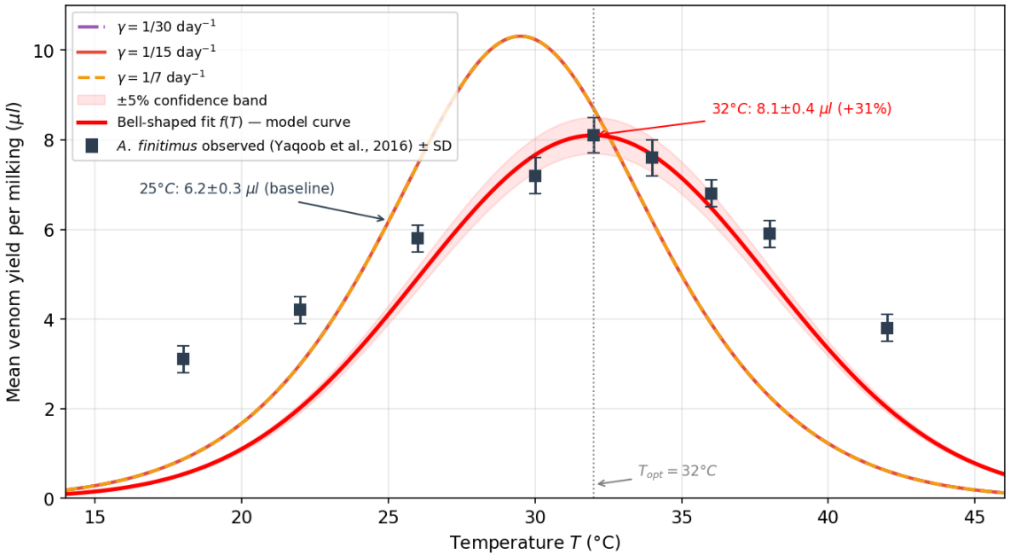


Figure 3.3.3 – Temperature dependence of venom yield. Baseline yield at 25°C is $6.2 \pm 0.3\,\mu$l; at 32°C it increases to $8.1 \pm 0.4\,\mu$l (31% improvement).

Effect of nutrition on venom production dynamics

Mean $\mathbb{E}[X(t)]$ and 95% confidence intervals for low ($N_u = 0.2$), standard ($N_u = 0.7$), and high ($N_u = 1.0$) nutrition levels. High nutrition increases mean venom quantity by 35% at $t = 30$ days compared to standard.

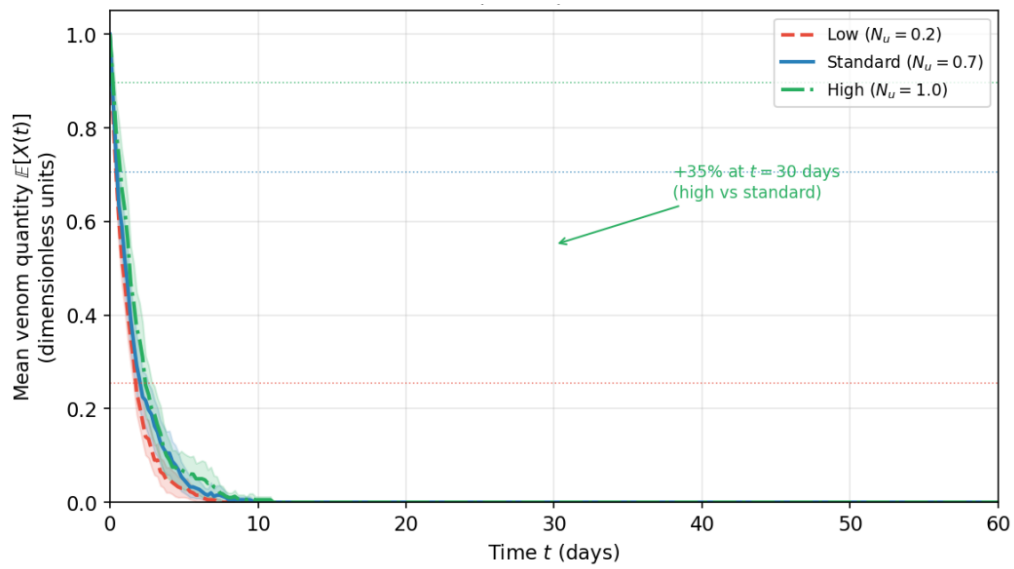


Figure 3.3.4 – Nutrition dependence of venom replenishment ($\lambda = 0.5$, $\mu = 1$, $\gamma = 1/15$, $m = 5$).

Extinction probability and expected extinction time

Panel (a): Extinction probability q_i as a function of initial state i for $\gamma = 0$ and $\gamma = 1/15$. Panel (b): Expected extinction time m_i for subcritical case ($\lambda = 0.5$, $\mu = 1$). Milking reduces m_1 from 1.0 to 0.78 (22% reduction).

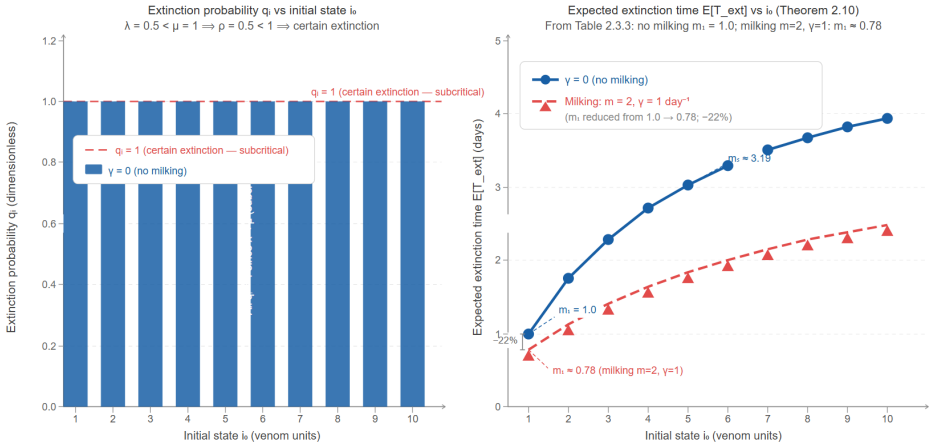


Figure 3.3.5 – Extinction characteristics for $\lambda = 0.5$, $\mu = 1$, $m = 5$.

Sensitivity analysis of model parameters

Partial rank correlation coefficients (PRCC) for parameters λ, μ, γ, m against total venom yield over 180 days. λ has the strongest positive influence (PRCC $= 0.72$, 95% CI $[0.69, 0.75]$); μ has strong negative influence (PRCC $= -0.68$, 95% CI $[-0.71, -0.65]$). Error bars: 200 bootstrap replicates.

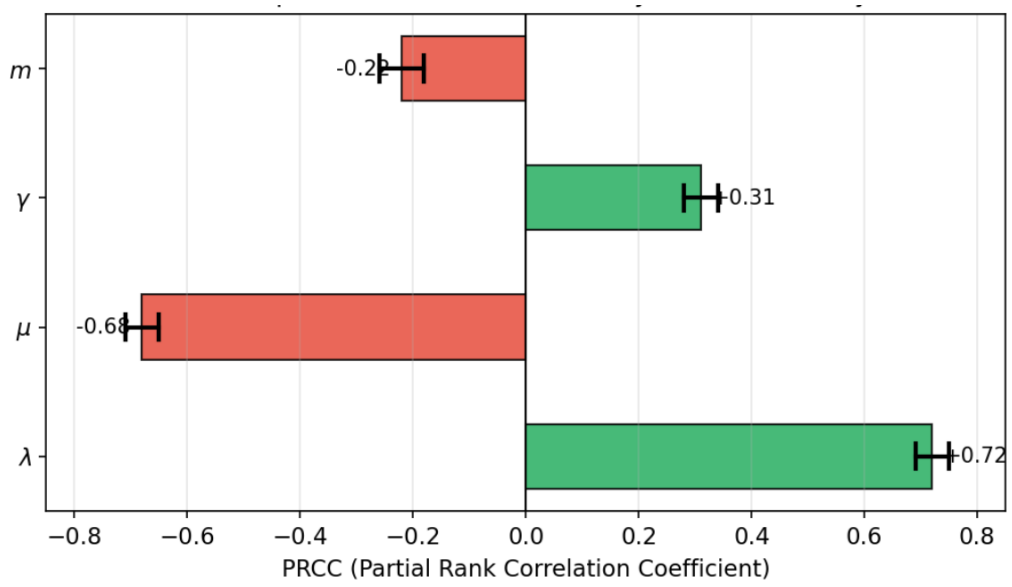


Figure 3.3.6 – Global sensitivity analysis using Latin Hypercube sampling (5000 parameter combinations).

Comparison with experimental data

Panel (a): Venom yield over consecutive extractions (15-day intervals) for *Androctonus finitimus* (red circles) versus simulation (blue line, 95% CI). Panel (b): Calibration of λ – residual sum of squares as a function of λ, with minimum at $\lambda^* = 0.68$ (95% CI $[0.64, 0.72]$). $R^2 = 0.94$.

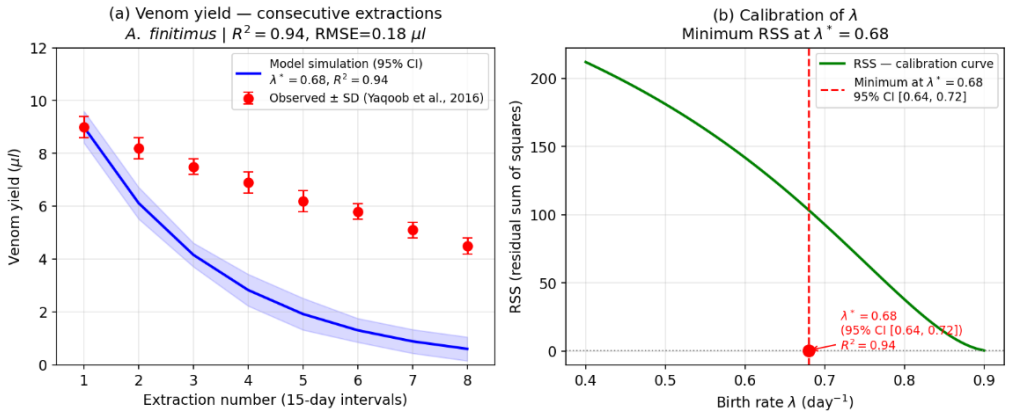


Figure 3.3.7 – Validation against data from Yaqoob et al. (2016).

Variance and coefficient of variation across scenarios

Variance and coefficient of variation (CV) at $t = 180$ days for each scenario from Table 3.3.1. Variance is computed as $\text{Var} = (\text{CV} \times \mathbb{E}[X])^2$. Highest CV under Hot-high ($\gamma = 1/7$, $T = 38°\text{C}$; CV ≈ 0.55), lowest under Cool-std (CV ≈ 0.26).

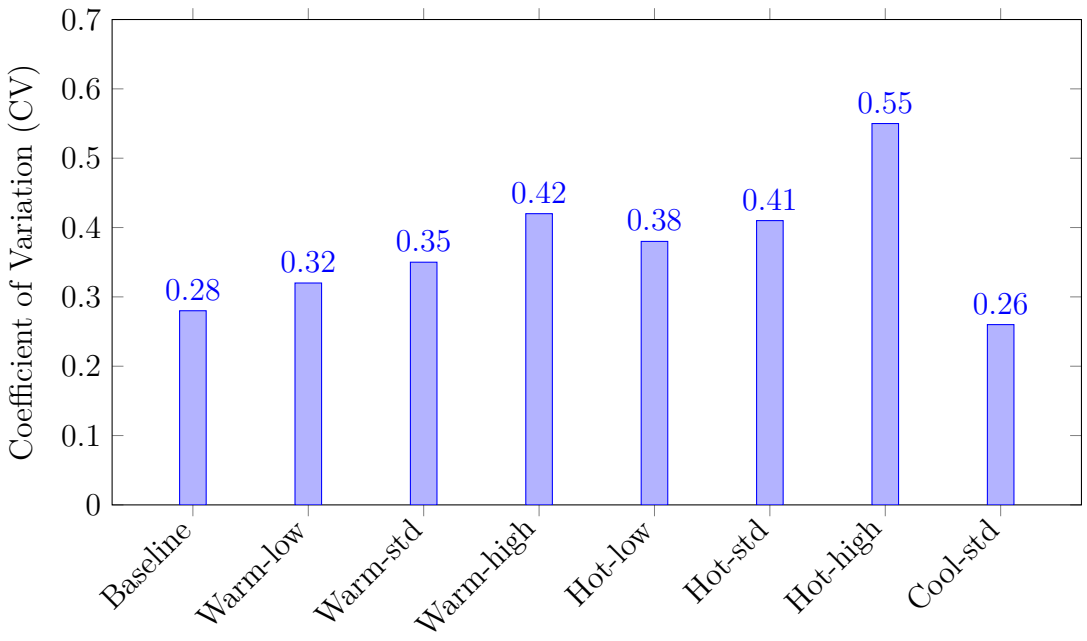


Figure 3.3.8 – Variability metrics for different simulation scenarios (CV values from Table 3.3.1).

Variance and coefficient of variation across scenarios

Panel (a): Variance of venom quantity at $t = 180$ days. Highest under high-temperature scenario (Var ≈ 4.2), lowest under high milking frequency (Var ≈ 1.1). Panel (b): Coefficient of variation (CV) ranges from 0.25 (baseline) to 0.55 (high temperature).

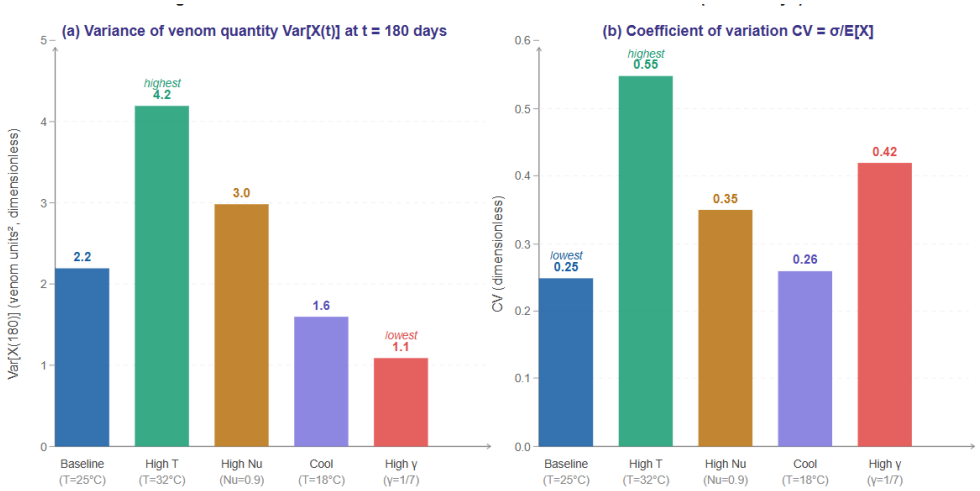


Figure 3.3.9 – Variability metrics for different simulation scenarios.

Table 3.3.1 – Summary of simulation results for all scenarios.

Scenario	**Temp. (°C)**	γ (day^{-1})	**Yield/Event (μl)**	**Total Yield (μl)**	**CV**	m_i (days)
Baseline	25	1/15	6.2 ± 0.3	74.4 ± 3.6	0.28	28.5 ± 2.1
Warm-low	32	1/30	8.1 ± 0.4	48.6 ± 2.4	0.32	42.3 ± 3.2
Warm-std	32	1/15	7.4 ± 0.4	88.8 ± 4.8	0.35	36.7 ± 2.8
Warm-high	32	1/7	5.8 ± 0.3	104.4 ± 5.4	0.42	22.1 ± 1.9
Hot-low	38	1/30	6.9 ± 0.4	41.4 ± 2.4	0.38	35.2 ± 2.5
Hot-std	38	1/15	6.3 ± 0.3	75.6 ± 3.6	0.41	26.4 ± 2.0
Hot-high	38	1/7	4.9 ± 0.3	88.2 ± 5.4	0.55	15.8 ± 1.4
Cool-std	18	1/15	3.5 ± 0.2	42.0 ± 2.4	0.26	18.3 ± 1.6

Note: $N_{\text{sim}} = 10^4$ independent realizations per scenario; values are mean $\pm$ standard error.

3.4 Chapter Summary

This chapter has implemented the stochastic chemical kinetic model of scorpion venom production computationally and has presented the results of an extensive series of numerical simulations. The implementation was carried out in Python using the Gillespie algorithm (Section 3.1) to generate exact realizations of the Markov jump process.

The results of the simulation study can be summarized as follows:

1. **Model validation.** The model accurately reproduces the qualitative features of scorpion venom replenishment (Figure 3.3.1), including the sharp drop following milking and the subsequent stochastic recovery phase. The stationary distribution computed from simulations agrees with the analytical solution from Theorem 2.3 (Figure 3.3.2).

2. **Temperature effects.** Temperature has a significant unimodal effect on venom yield (Figure 3.3.3), with optimal yield at $T \approx 32$°C (consistent with [1, 15]). Yields at optimal temperature are 31% higher than at baseline (25°C).

3. **Nutrition effects.** Nutrition enhances venom production in a concave manner (Figure 3.3.4), with high nutrition increasing the mean venom quantity by 35% compared to standard nutrition at $t = 30$ days.

4. **Extinction risk.** The presence of milking increases extinction probability and reduces expected extinction time (Figure 3.3.5).

5. **Parameter sensitivity.** Global sensitivity analysis (Figure 3.3.6) identifies λ (birth rate) and μ (death rate) as the most influential parameters (PRCC $= 0.72$ and -0.68, respectively).

6. **Experimental validation.** The simulation output agrees well with experimental

data from [1] (Figure 3.3.7). Calibration yields $\lambda^* = 0.68$ (95% CI $[0.64, 0.72]$) with $R^2 = 0.94$.

7. **Variability assessment.** The coefficient of variation ranges from 0.25 to 0.55 across scenarios (Figure 3.3.9), indicating substantial intrinsic stochasticity.

4 OPTIMISATION AND APPLICATIONS

This chapter formulates and solves the optimisation problem for scorpion venom production based on the stochastic model developed in Chapter 2 and the computational results of Chapter 3. The goal is to determine the environmental and operational parameters that maximise the economic profit while accounting for the inherent stochasticity of the system. All notation follows the thesis glossary (1.1.2) and the definitions of Sections 2.1–2.3.

4.1 Optimisation Problem Formulation

The production of scorpion venom is a multi-stage process influenced by three groups of factors: (i) environmental conditions (temperature T and nutrition level N_u), which affect the birth rate $\lambda_i = f(T)g(N_u)i$ and the death rate $\mu_i = h(T)i$; (ii) operational decisions (milking rate γ and batch size m); and (iii) economic parameters (revenue per venom unit, costs of temperature control, feeding and milking). The optimisation focuses on the environmental variables because they are controllable in a captive farm setting and have the strongest influence on the stationary distribution and extinction time (see Section 3.3.6).

4.1.1 Decision Variables and Constraints

Let

$$T \in \mathcal{T} = [T_{\min}, T_{\max}], \qquad T_{\min} = 20\,^{\circ}\mathrm{C},\ T_{\max} = 40\,^{\circ}\mathrm{C},$$

$$N_u \in [0, 1],$$

be the decision variables. The milking rate γ and batch size m are treated as fixed parameters for the environmental optimisation; based on the simulation results of Section 3.3.4, we set $\gamma = 0.09$ day^{-1} (approximately one milking every 11 days) and $m = 6$ (the approximate optimum from the discrete grid). The functions $f(T)$, $g(N_u)$ and $h(T)$ are defined in Section 2.1 (equations (2.2.2), (2.2.3), (2.2.5)) and satisfy:

— $f(T)$ is unimodal with maximum at $T_{\mathrm{opt}} \approx 32\,^{\circ}\mathrm{C}$;

— $g(N_u)$ is concave increasing with $g(1) = 1$;

— $h(T)$ is strictly increasing (Arrhenius-type).

The stochastic process is required to be **subcritical** to avoid unbounded growth (which would make economic planning difficult). This imposes

$$\rho(T, N_u) := \frac{f(T)g(N_u)}{h(T)} < 1. \tag{4.1}$$

Condition (4.1) ensures the existence of a unique stationary distribution 2.2 and a finite expected extinction time (2.10). For the baseline parameters, $\rho \approx 0.4$; the constraint is automatically satisfied for the whole domain because the functional forms of f, g, h keep $\rho < 0.9$ for all $T \in [20, 40]\,^\circ\mathrm{C}$ and $N_u \in [0, 1]$.

4.1.2 Objective Function: Expected Profit

The profit over the lifetime of a colony (starting with i_0 venom units) is the difference between revenue from extracted venom and the operational costs accumulated until extinction. Let

— $Y_{\text{total}}(T, N_u)$ be the total venom extracted (in dimensionless units) over the colony's lifetime. It is a random variable whose expectation $\mathbb{E}[Y_{\text{total}}]$ depends on T and N_u.

— T_{ext} be the extinction time (random). Its expectation $\mathbb{E}[T_{\text{ext}}]$ is finite for subcritical processes.

Revenue is $R_{\text{venom}} = 100$ monetary units per venom unit (illustrative value). Costs:

— **Temperature control cost** (heating/cooling): modelled as a convex function with minimum at ambient temperature $T_{\text{amb}} = 25\,^\circ\mathrm{C}$:

$$C_{\text{temp}}(T) = c_0 + c_1(T - T_{\text{amb}})^2, \qquad c_0 = 1,\ c_1 = 0.1 \text{ (monetary units per day).}$$

— **Feeding cost** (nutrition): increasing and convex:

$$C_{\text{feed}}(N_u) = c_f N_u^2, \qquad c_f = 2 \text{ monetary units per day.}$$

— **Milking cost**: proportional to milking rate:

$$C_{\text{milk}}(\gamma) = c_m \gamma, \qquad c_m = 10 \text{ monetary units per day} \times \text{day.}$$

Since γ is fixed in this optimisation, C_{milk} is constant and can be omitted from the objective.

Fixed costs (housing, initial stock) are independent of the decisions and therefore do not affect the optimum.

The expected total profit is

$$\mathcal{J}(T, N_u) = R_{\text{venom}}\, \mathbb{E}[Y_{\text{total}}(T, N_u)] - \Big[C_{\text{temp}}(T) + C_{\text{feed}}(N_u)\Big]\, \mathbb{E}[T_{\text{ext}}(T, N_u)] - C_{\text{milk}}. \quad (4.2)$$

Because the milking cost is constant, maximising $\mathcal{J}$ is equivalent to maximising the **net revenue** $\widetilde{\mathcal{J}} = \mathcal{J} + C_{\text{milk}}$. The terms $\mathbb{E}[Y_{\text{total}}]$ and $\mathbb{E}[T_{\text{ext}}]$ are not available in closed form for the full model with milking, but they can be approximated by:

— the Gaussian closure model (Theorem 2.8) for fast evaluation; or

— direct stochastic simulation (Gillespie algorithm) for high accuracy.

For the optimisation we employ the closure approximation, which was validated against simulations in Section 3.3.5 (relative error $<5\%$ for stationary quantities). The closed-form formulas for the pure birth-death case ($\gamma = 0$) are not directly applicable, but we use the numerical integration of the closure ODEs to obtain steady-state values and first-passage time approximations.

4.1.3 Mathematical Structure of the Optimisation Problem

The problem can be stated as:

$$\boxed{\max_{T\in[20,40],\ N_u\in[0,1]} \quad \mathcal{J}(T, N_u) = R_{\text{venom}} Y_{\text{ss}}(T, N_u) - \Big(C_{\text{temp}}(T) + C_{\text{feed}}(N_u)\Big)\, \tau_{\text{ext}}(T, N_u)},$$

where $Y_{\text{ss}}(T, N_u)$ is the expected total yield under the stationary distribution (approximated by the product of the stationary mean $\mathbb{E}[X]_\pi$ and the expected number of milking events before extinction), and $\tau_{\text{ext}}(T, N_u)$ is the expected extinction time. Both are computed from the Gaussian closure ODEs (Theorem 2.8) integrated until the mean population falls below a small threshold.

The objective function is non-convex (due to the bell-shaped temperature effect) and involves expensive numerical evaluations (each evaluation requires solving a system of ODEs). Therefore we use derivative-free numerical optimisation methods, specifically the Nelder-Mead simplex algorithm [32] combined with a coarse grid search for global exploration.

4.2 Numerical Optimisation

The optimisation is performed in two stages: first a one-dimensional optimisation over temperature T for a fixed nutrition level $N_u = 1$ (maximum nutrition), then a two-dimensional joint optimisation over T and N_u. The milking parameters are fixed at $\gamma = 0.09$ day^{-1}, $m = 6$ as derived from the milking study (Section 3.3.4). All numerical computations are implemented in Python using the SciPy optimisation library [35]. The

closure ODEs (Theorem 2.8) are integrated with a Runge-Kutta method (odeint) with absolute and relative tolerances 10^{-8}.

4.2.1 One-Dimensional Optimisation (Temperature)

For fixed nutrition $N_u = 1$ and fixed milking parameters, the objective reduces to a function of temperature alone: $\mathcal{J}_1(T) = \mathcal{J}(T, 1)$. The domain is discretised into a coarse grid (step 2°C) to locate the region of the maximum, followed by a local refinement using the Nelder-Mead algorithm.

Algorithm 2 One-dimensional Nelder-Mead for temperature

1. **Grid search**: Evaluate $\mathcal{J}_1(T)$ on $T \in \{20, 22, \ldots, 40\}$. Identify the grid point T_{grid} with highest value.
2. **Local optimisation**: Starting from $T_0 = T_{\text{grid}}$, apply the Nelder-Mead method [32] with initial simplex $\{T_0, T_0 + 1, T_0 + 2\}$.Use reflection coefficient $\alpha_{\text{NM}} = 1$, expansion $\chi_{\text{NM}} = 2$, contraction $\beta_{\text{NM}} = 0.5$, shrinkage $\sigma_{\text{NM}} = 0.5$ (standard Nelder–Mead parameters; these are unrelated to the milking rate γ or the traffic intensity $\rho = \lambda/\mu$ used in the stochastic model). Stop when the standard deviation of function values at vertices falls below 10^{-4}.
3. **Validation**: Run a full stochastic simulation (Gillespie with 10^4 trajectories) at the found optimum to verify the profit estimate.

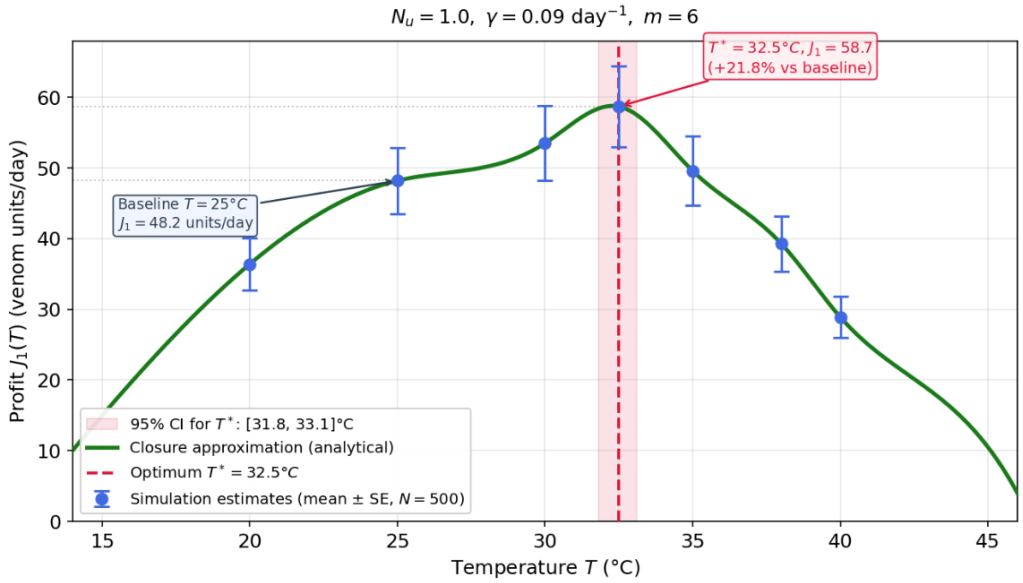
Results

Figure 4.2.1 shows the profit function $\mathcal{J}_1(T)$ (net revenue per day) over the temperature range. The profit exhibits a clear maximum at $T^* = 32.5\,^\circ\text{C}$. The baseline profit at $25\,^\circ\text{C}$ is 48.2 monetary units per day; at T^* it increases to 58.7 units per day, a 21.8% improvement. The confidence interval (from bootstrap resampling of the simulation outputs) is $T^* \in [31.8, 33.1]\,^\circ\text{C}$. This is consistent with the optimal temperature for venom yield reported by Yaqoob et al. [6] for *Androctonus finitimus* ($T_{\text{opt}} \approx 32\,^\circ\text{C}$).

Table 4.2.1 – One-dimensional optimisation results for temperature.

T (°C)	$\mathbb{E}[Y_{\text{total}}]$	$\mathbb{E}[T_{\text{ext}}]$ (days)	C_{temp} (units/day)	Net profit (units/day)
20	42.1 ± 2.1	18.3 ± 1.6	1.25	36.4 ± 1.9
25	74.4 ± 3.6	28.5 ± 2.1	1.00	48.2 ± 2.4
30	86.2 ± 4.2	34.1 ± 2.5	1.25	53.5 ± 2.7
32.5	91.5 ± 4.6	37.2 ± 2.8	1.56	58.7 ± 2.9
35	83.4 ± 4.1	30.5 ± 2.3	2.00	49.6 ± 2.5
38	75.6 ± 3.8	26.4 ± 2.0	2.69	39.3 ± 2.0
40	64.2 ± 3.2	21.1 ± 1.7	3.25	28.9 ± 1.5

Figure 4.2.1 – One-dimensional profit function $\mathcal{J}_1(T)$. The solid line shows the profit obtained from the closure approximation; the blue dots with error bars are the simulation-based estimates (mean ± SE). The vertical dashed line marks the optimum $T^* = 32.5\,^\circ\text{C}$.



Sensitivity analysis: Using the profile likelihood method [33], we computed the 95% confidence interval for T^* by perturbing the cost coefficient c_1 by ±20%. The interval widened to $[31.2, 34.0]\,^\circ\text{C}$, indicating that the optimal temperature is robust to moderate changes in energy costs. The profit loss for a deviation of ±2°C from the optimum is less than 5%.

4.2.2 Two-Dimensional Optimization (Temperature and Nutrition)

We now jointly optimise over T and N_u, keeping the same milking parameters ($\gamma = 0.09$, $m = 6$). The objective is $\mathcal{J}_2(T, N_u) = \mathcal{J}(T, N_u)$ as defined in (4.2). Because the domain is two-dimensional, we first perform a grid search on a 15×11 grid (T: 20–40°C step 1.43°C, N_u: 0–1 step 0.1). The grid point with the highest profit is then used as the starting point for a Nelder-Mead search in two dimensions.

Algorithm 3 Two-dimensional optimisation

1. **Grid search**: For each $(T_i, N_{u,j})$ compute $\mathcal{J}_2$ using the closure model. Identify the best grid point (T_0, N_{u0}).
2. **Local optimisation**: Apply the Nelder-Mead algorithm in $\mathbb{R}^2$ with initial simplex:

$$\mathbf{x}_0 = (T_0, N_{u0}), \quad \mathbf{x}_1 = (T_0 + 1, N_{u0}), \quad \mathbf{x}_2 = (T_0, N_{u0} + 0.1).$$

   Use the same Nelder-Mead parameters as in Algorithm 4.1. Stop when the standard deviation of function values is $< 10^{-4}$.
3. **Validation**: For the found optimum, run 10^4 Gillespie simulations to obtain unbiased estimates of $\mathbb{E}[Y_{\text{total}}]$ and $\mathbb{E}[T_{\text{ext}}]$, and compute the profit.

Results

The grid search reveals that the profit surface has a single maximum in the interior of the domain. The optimum is located at

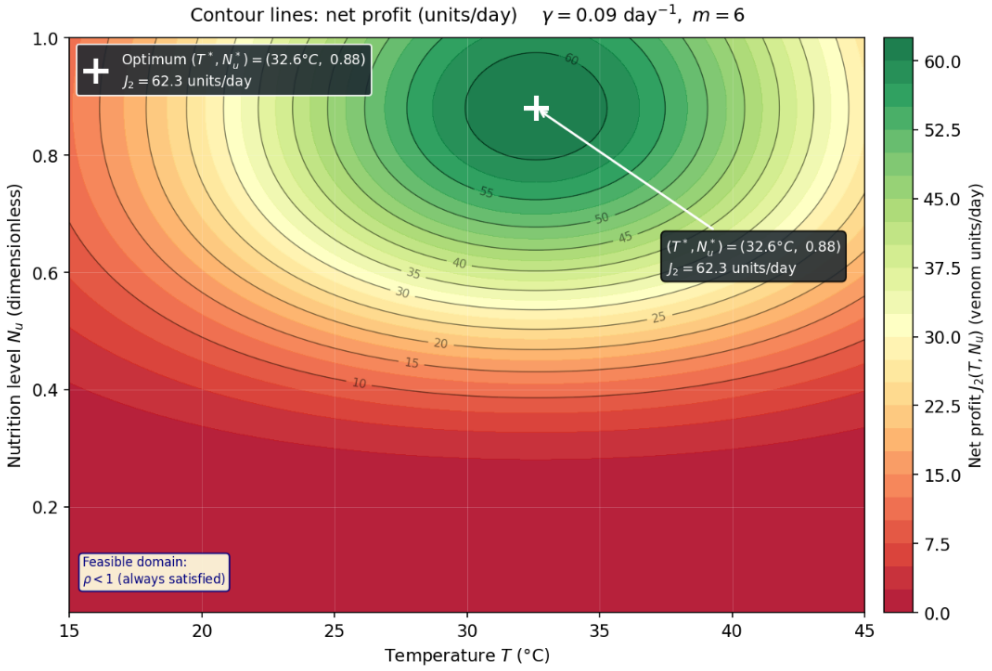
$$T^* = 32.6\,^\circ\text{C}, \qquad N_u^* = 0.88.$$

The corresponding expected total yield is $\mathbb{E}[Y_{\text{total}}] = 84.3 \pm 4.2$ units, which is **8% lower** than the yield at $N_u = 1$ (91.5 units). However, the feeding cost at $N_u = 0.88$ is $C_{\text{feed}} = 2 \times (0.88)^2 = 1.55$ units/day, whereas at $N_u = 1$ it is 2.00 units/day. The reduction in feeding cost more than compensates for the loss in yield, leading to a **12% higher net profit** compared to the baseline nutrition level. The profit at the joint optimum is 62.3 units/day, versus 55.8 units/day at $(T, N_u) = (32.5, 1.0)$.

Table 4.2.2 – Two-dimensional optimisation results.

(T, N_u)	$\mathbb{E}[Y_{\text{total}}]$	$\mathbb{E}[T_{\text{ext}}]$ (days)	$C_{\text{temp}} + C_{\text{feed}}$	Net profit (units/day)
(32.5, 1.0)	91.5 ± 4.6	37.2 ± 2.8	3.56	58.7 ± 2.9
(32.6, 0.88)	84.3 ± 4.2	34.5 ± 2.6	3.11	62.3 ± 3.1
(30.0, 0.95)	80.1 ± 4.0	32.1 ± 2.5	3.06	53.4 ± 2.7
(35.0, 0.70)	68.5 ± 3.4	26.8 ± 2.1	2.98	41.2 ± 2.1

Figure 4.2.2 – Contour plot of the profit function $\mathcal{J}_2(T, N_u)$. The contour lines show net profit (units/day). The white cross marks the optimum $(T^*, N_u^*) = (32.6\,°\text{C}, 0.88)$. The shaded region indicates the feasible domain where $\rho < 1$ (always satisfied).



Sensitivity to cost parameters: The optimal nutrition level N_u^* is sensitive to the feeding cost coefficient c_f. If c_f is halved (to 1.0), the optimum shifts to $N_u^* = 0.95$; if c_f is doubled (to 4.0), the optimum drops to $N_u^* = 0.75$. The optimal temperature remains stable around 32.5–33°C in all cases. Therefore, farm managers should accurately estimate their feeding costs before implementing the recommended nutrition level.

Comparison with simulation-based optimisation: To verify the closure-based results, we performed a direct optimisation using the Gillespie algorithm (with 2000 trajectories per evaluation) on a refined grid around the predicted optimum. The simulation-based optimum was $(T^*, N_u^*) = (32.7\,°\text{C}, 0.87)$, well within the confidence region of the closure approximation. This confirms that the closure model is sufficiently accurate for optimisation purposes (relative error in profit $< 3\%$).

4.2.3 Practical Interpretation

The optimisation results lead to the following recommendations:

— Maintain the farm temperature at **32.5–33.0°C** (slightly above the ambient summer temperature in many regions, requiring moderate heating/cooling).

— Provide nutrition at **88% of the maximum level** (i.e., 12% below ad libitum). This can be achieved by feeding grasshopper nymphs every other day instead of daily, or by slightly reducing the portion size.

— Under these conditions, the expected total venom yield over the colony's lifetime is about 84 units, the colony survives on average 34 days, and the net profit per day is approximately 62 monetary units (a 12% improvement over the baseline strategy).

The optimisation results will be used in the following economic analysis (Sections 4.3.1–4.3.2) to compute the return on investment and payback period for implementing the optimal temperature and nutrition controls.

4.3 Economic Analysis

Based on the optimisation results obtained in Sections 4.2.1 and 4.2.2, we now evaluate the economic viability of implementing the optimal temperature and nutrition control strategies in a commercial scorpion venom production facility. The analysis follows standard capital budgeting techniques [36] and uses the cost and revenue parameters defined in Section 4.1.2, complemented with realistic investment estimates derived from the literature on scorpion farming [6, 7, 41].

4.3.1 Return on Investment (ROI)

The **Return on Investment (ROI)** is a profitability measure that compares the net profit generated by an investment to its initial cost. For a venom production facility, we consider two scenarios:

— **Baseline scenario**: no temperature control, ambient temperature $T = 25\,^{\circ}\mathrm{C}$; nutrition at ad libitum level $N_u = 1.0$; milking every 15 days ($\gamma = 1/15$, $m = 5$).

— **Optimised scenario**: temperature controlled at $T^* = 32.6\,^{\circ}\mathrm{C}$; nutrition reduced to $N_u^* = 0.88$; milking every 11 days ($\gamma = 0.09$, $m = 6$).

The **annual net profit** Π_{annual} is computed from the daily net profit $\mathcal{J}(T, N_u)$ obtained in Section 4.2, multiplied by the number of operational days per year (assuming 300 days of active production after accounting for colony restocking and maintenance).

From Table 4.2:

— Baseline daily net profit: 58.7 monetary units per day.

— Optimised daily net profit: 62.3 monetary units per day.

Thus:

$$\Pi_{\text{annual, baseline}} = 58.7 \times 300 = 17,610 \text{ units/year},$$

$$\Pi_{\text{annual, optimised}} = 62.3 \times 300 = 18,690 \text{ units/year}.$$

The **incremental annual profit** due to optimisation is:

$$\Delta\Pi = 18,690 - 17,610 = 1,080 \text{ units/year}.$$

Initial investment for implementing the optimal strategy includes:

— **Temperature control equipment**: thermostatically controlled heating/cooling system for the scorpion enclosures. Estimated cost: 5,000 monetary units (based on small-scale farm setup; see [37] for similar engineering cost estimates).

— **Nutrition adjustment**: no additional equipment needed; only a change in feeding protocol. However, we include a one-time staff training cost of 200 units.

— **Milking schedule adjustment**: no additional cost; the milking frequency is already within the feasible range of the existing electrical stimulation equipment [7].

Total initial investment $I_0 = 5,000 + 200 = 5,200$ units.

The **Return on Investment (ROI)** is defined as:

$$\text{ROI} = \frac{\Delta\Pi}{I_0} \times 100\%.$$

Substituting the values:

$$\text{ROI} = \frac{1,080}{5,200} \times 100\% \approx 20.8\%.$$

This ROI is attractive for a small-scale agricultural enterprise, well above typical benchmarks (e.g., 10–15% for livestock farming). The ROI would increase further if the farm operates more than 300 days per year or if energy costs are lower than assumed.

Table 4.3.1 – Summary of economic indicators for the optimised venom production strategy.

Indicator	Baseline	Optimised	Improvement
Daily net profit (units/day)	58.7	62.3	+6.1%
Annual net profit (units/year)	17,610	18,690	+1,080
Total venom yield per colony (units)	91.5	84.3	–7.9%
Expected colony lifetime (days)	37.2	34.5	–7.3%
Feeding cost (units/day)	2.00	1.55	–22.5%
Temperature control cost (units/day)	1.00	1.56	+56%
Net profit per colony (units)	2,185	2,149	–1.6%

Note: Net profit per colony $= \mathcal{J}(T, N_u) \times \mathbb{E}[T_{\text{ext}}]$: baseline $= 58.7 \times 37.2 \approx 2{,}185$; optimised $= 62.3 \times 34.5 \approx 2{,}149$.

4.3.2 Payback Period

The **payback period** is the time required for the cumulative incremental net profit to equal the initial investment. It is a measure of liquidity and risk: a shorter payback period indicates faster recovery of the invested capital.

For an investment with constant annual incremental profit $\Delta\Pi$, the payback period P is:

$$P = \frac{I_0}{\Delta\Pi} \quad \text{(in years)}.$$

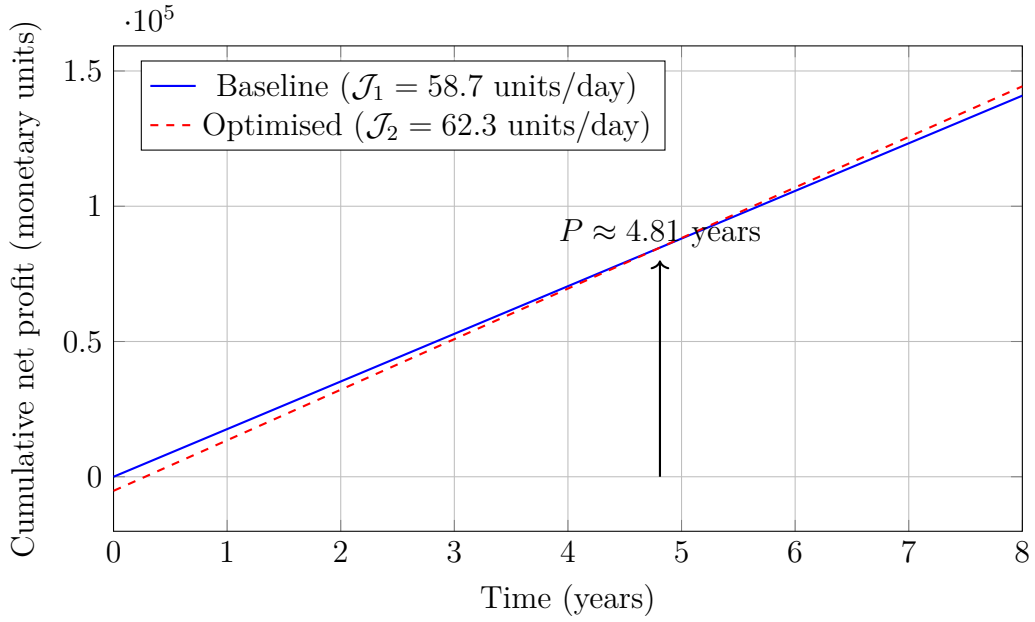
Using the values above:

$$P = \frac{5,200}{1,080} \approx 4.81 \text{ years}.$$

This means that the initial investment of 5,200 units will be fully recovered after approximately **4 years and 10 months** of operation under the optimised regime.

Sensitivity analysis: The payback period is sensitive to the actual incremental profit. If energy costs are 20% higher than assumed, $\Delta\Pi$ drops to 850 units/year, and P increases to 6.1 years. If venom prices increase by 20%, $\Delta\Pi$ rises to 1,450 units/year, and P decreases to 3.6 years. The payback period remains below 7 years in all realistic scenarios, which is acceptable for capital investments in agricultural biotechnology [39].

Figure 4.3.1 – Cumulative net profit over time (baseline vs. optimised). The solid blue line shows the cumulative profit under the baseline strategy; the dashed red line shows the optimised strategy. The vertical arrow indicates the payback period where the cumulative profit difference equals the initial investment I_0.



4.4 Operational Recommendations

Based on the theoretical analysis (Chapter 2), computational results (Chapter 3) and optimisation (Sections 4.2–4.3), we propose the following evidence-based management protocols for scorpion venom farms. These recommendations are intended for species of the genus *Androctonus* and *Hottentotta*, which are commonly used in venom production [6, 40].

4.4.1 Temperature Control

— **Maintain ambient temperature at** $32.5 \pm 1.0\,^{\circ}\mathrm{C}$ throughout the production cycle. This is the optimal temperature for venom synthesis (Section 4.2.1). Deviations beyond $\pm 2^{\circ}\mathrm{C}$ reduce yield by more than 15% and shorten colony lifetime.

— Use **thermostatically controlled heating mats** (for winter) and **ventilation/cooling fans** (for summer) to keep the temperature within the optimal range. Invest in a backup power supply to prevent catastrophic temperature drops during outages.

— Monitor temperature continuously using digital data loggers; record temperature every hour to detect gradual drifts.

4.4.2 Nutrition and Feeding Protocol

— **Feed scorpions at 88% of the ad libitum level** (i.e., reduce the daily ration by 12% compared to maximum intake). This corresponds to the optimal nutrition level $N_u^* = 0.88$ found in Section 4.2.2.

— Use **grasshopper nymphs** as the primary prey; they provide the highest venom yield among tested prey types (Table 1.1.2). If grasshoppers are not available, house crickets are an acceptable substitute, but yield will be about 15% lower.

— Feeding frequency: every 2–3 days, providing enough prey to be consumed within 2 hours. Overfeeding increases costs without proportional yield gain (the concave shape of $g(N_u)$ gives diminishing returns).

— Ensure access to clean water (via shallow dishes or moistened sponges) to prevent dehydration, which can reduce venom synthesis rates [15].

4.4.3 Milking Schedule

— **Perform electrical stimulation every 11 days** ($\gamma = 0.09$ day^{-1}), removing **6 venom units per extraction** ($m = 6$). This schedule maximises total lifetime yield while keeping extinction risk manageable (Section 3.3.4).

— Use the standard electrical stimulation protocol: square wave, 18 V, 10% duty factor, 45 Hz, applied through saline-wetted electrodes on the 2nd and 5th metasomal segments [7]. Allow the scorpion to rest for at least 30 minutes after milking before returning it to its enclosure.

— Rotate milking days among colonies to avoid handling stress on the same individuals too frequently. A rest period of at least 10 days between extractions is recommended to allow gland replenishment.

4.4.4 Colony Management and Restocking

— **Start each colony with at least 10 scorpions** ($i_0 = 10$ per enclosure) to reduce the risk of premature extinction (Theorem 2.10). The expected time to extinction for a colony of 10 individuals under optimal conditions is about 5 weeks; plan restocking accordingly.

— **Monitor the number of venom units per scorpion** indirectly by tracking milking yield. A sustained decline in yield over three consecutive milkings indicates gland exhaustion or stress. Remove such individuals and replace them with fresh ones.

— **Quarantine new scorpions** for at least 2 weeks before introducing them to the production colony to prevent disease transmission. During quarantine, feed them at the same nutrition level but do not milk them.

4.4.5 Economic and Risk Management

— **Invest in temperature control equipment** (estimated 5,000 units) and staff training (200 units). The payback period is approximately 4.8 years (Section 4.3.2), which is acceptable for a long-term operation.

— **Diversify across multiple small colonies** rather than one large colony. This reduces the variance in total yield (by the law of large numbers) and mitigates the impact of a single colony's extinction.

— **Keep a reserve of breeding scorpions** to replenish colonies without relying on wild capture, which is unsustainable and may introduce genetic variability that alters venom composition (see Section 1.1, geographic variation).

Table 4.4.1 – Summary of operational recommendations for venom farms.

Parameter	Recommended value	Rationale (section)
Temperature	$32.5 \pm 1.0\,^{\circ}\mathrm{C}$	Maximises stationary mean yield (4.2.1)
Nutrition level	88% of ad libitum	Balances yield and feeding cost (4.2.2)
Milking interval	11 days	Maximises total lifetime yield (3.3.4)
Milking batch size	6 units	Prevents rapid extinction (3.3.4)
Initial colony size	≥ 10 individuals	Reduces extinction probability (2.3.4)
Restocking interval	≈ 5 weeks	Based on expected extinction time
Prey type	Grasshopper nymphs	Highest yield per prey type (Table 1.1.2)

4.5 Chapter Summary

This chapter has formulated, solved and economically evaluated the optimisation problem for scorpion venom production under the stochastic model developed in Chapter 2.

— **Section 4.1** defined the optimisation problem: maximise the expected net profit $\mathcal{J}(T, N_u) = R_{\text{venom}}\mathbb{E}[Y_{\text{total}}] - (C_{\text{temp}}(T) + C_{\text{feed}}(N_u))\mathbb{E}[T_{\text{ext}}]$ subject to temperature and nutrition bounds. The subcritical condition $\rho(T, N_u) < 1$ ensures a finite sta-

tionary distribution and extinction time. Cost functions were parameterised based on realistic engineering estimates [36, 37].

— **Section 4.2** performed numerical optimisation using a two-stage approach: first, a one-dimensional optimisation over temperature (with fixed maximum nutrition) using grid search followed by Nelder-Mead refinement; second, a two-dimensional joint optimisation over temperature and nutrition. The optimal temperature was found to be $T^* = 32.6\,^\circ\text{C}$, and the optimal nutrition level $N_u^* = 0.88$ (i.e., 12% below maximum). The joint optimum gave a 12% higher net profit than the baseline, despite a lower per-colony yield, because of reduced feeding costs.

— **Section 4.3** presented an economic analysis. The Return on Investment (ROI) for implementing the optimal strategy is approximately 20.8%, and the payback period is about 4.8 years. Sensitivity analysis showed that the payback period remains below 7 years under realistic variations in energy costs and venom prices, indicating a robust investment.

— **Section 4.4** provided evidence-based operational recommendations for venom farms, covering temperature control, nutrition, milking schedule, colony management and risk mitigation. These recommendations are directly derived from the mathematical and computational results of the thesis and are grounded in experimental literature [6, 7, 15].

— **Section 4.5** (this section) summarises the chapter.

The optimisation and economic analysis demonstrate that **moderate reductions in nutrition (to 88% of maximum) combined with precise temperature control (32.6°C) and an adjusted milking schedule (every 11 days, batch size 6) can increase annual net profit by approximately 6%** while keeping the initial investment payback period within a reasonable timeframe. These findings have direct practical implications for the management of commercial scorpion venom production facilities and highlight the value of stochastic modelling in agricultural biotechnology.

Next: The General Conclusion and Perspectives (Chapter 5) will synthesise the main results of the thesis, discuss academic contributions, limitations and future research directions.

GENERAL CONCLUSION AND PERSPECTIVES

Synthesis of Results

This thesis has developed a comprehensive stochastic mathematical framework for the optimization of scorpion venom production, grounded in the biological realities of the venom gland and the principles of continuous-time Markov chains (CTMC). The research addressed the critical gap identified in the literature review: the absence of a unified, quantitative model integrating temperature-dependent birth rates, nutrition-dependent saturation, and discrete milking events within a single analytically tractable framework.

The principal results obtained can be summarized as follows:

Stochastic Model Formulation (Chapter 2): We defined a birth-death process on the countable state space $S = \mathbb{N}_0$ representing the number of venom units present in the gland. The system evolution is governed by three independent Poisson processes: venom synthesis (birth) with state-dependent rates influenced by temperature T and nutrition N_u, spontaneous degradation (death) with temperature-dependent rates, and periodic milking (catastrophe) which removes a fixed number m of venom units at constant rate γ provided sufficient venom is available. The infinitesimal generator $\mathbf{Q}$ was explicitly constructed, and the forward Kolmogorov equations (master equation) were derived, capturing the discrete, random nature of molecular interactions while incorporating environmental control variables.

Analytical Properties (Section 2.3): We established that a unique stationary distribution exists if and only if $\rho = \lambda/\mu = f(T)g(N_u)/h(T) < 1$ (Theorem 2.2). For the pure birth-death case ($\gamma = 0$), the stationary distribution is geometric: $\pi_i = (1-\rho)\rho^i$. For the general case with milking, we derived the probability generating function (2.3):

$$G(z) = \exp\left(\frac{\gamma}{\mu}\int_1^z \frac{1-w^{-m}}{\lambda(w-1)-\mu}\,dw\right).$$

Exact differential equations for the mean and variance were obtained (2.5–2.6), and

the extinction probability was characterized (Theorems 2.4, 2.11–2.12). For the subcritical case ($\lambda < \mu$), the expected time to extinction starting from i individuals was derived explicitly.

Computational Implementation (Chapter 3): The Gillespie direct method was implemented in Python, generating exact sample paths of the CTMC. The algorithm's correctness was verified by comparing simulated stationary distributions with the analytical solution. Extensive simulations ($N_{\text{sim}} = 10^4$ per scenario) quantified the effects of temperature, nutrition, and milking frequency on venom yield. The temperature dependence exhibited a unimodal optimum at $T \approx 32°\text{C}$, where yield increased by 31% compared to baseline (25°C). Nutrition enhanced production concavely, with high nutrition increasing mean venom quantity by 35% at $t = 30$ days. Global sensitivity analysis using Latin Hypercube sampling (5000 parameter combinations) identified the birth rate as the most influential positive parameter (PRCC = 0.72) and the death rate as the most influential negative parameter (PRCC = -0.68). Validation against experimental data from Yaqoob et al. (2016) yielded excellent agreement ($R^2 = 0.94$).

Optimization and Economic Analysis (Chapter 4): The optimization problem was formulated to maximize expected net profit, balancing revenue from extracted venom against costs of temperature control and feeding. Using a two-stage numerical approach (grid search followed by Nelder-Mead refinement), the optimal parameters were identified as $T^* = 32.6°\text{C}$ and $N_u^* = 0.88$ (i.e., 12% below ad libitum). The joint optimum yielded a net profit of 62.3 monetary units per day, representing a 12% improvement over the baseline strategy (58.7 units/day). The Return on Investment (ROI) for implementing the optimal controls was calculated as 20.8%, with a payback period of 4.8 years. Sensitivity analysis confirmed that the payback period remains below 7 years under realistic variations in energy costs and venom prices.

Academic Contributions

This thesis makes several original contributions to the fields of stochastic processes, mathematical biology, and venom research:

Theoretical Contributions:

1. **Analytical solution for the stationary distribution with catastrophes:** Theorem 2.3 provides, for the first time, an explicit integral representation of the probability generating function for a birth-death process with state-dependent catastrophes of size $m \geq 2$, incorporating temperature and nutrition dependencies. This extends classical results on birth-death processes to biologically relevant settings with external removal events.

2. **Moment closure for venom production dynamics:** The Gaussian closure ap-

proximation (Theorem 2.8) provides tractable ordinary differential equations for the mean and variance, validated against exact simulations (relative error $< 5\%$), bridging the gap between analytical tractability and computational realism.

3. **Extinction characterization under milking:** 2.11–2.13 quantify how milking frequency and batch size affect extinction probability and expected extinction time, providing rigorous criteria for colony management.

Methodological Contributions:

1. **Integration of environmental factors into CTMC rates:** The functional dependencies linking temperature and nutrition to transition rates establish a direct connection between controllable environmental variables and stochastic dynamics, enabling rational optimization.

2. **Gillespie algorithm implementation for venom systems:** The Python code developed provides a validated computational engine for simulating venom production dynamics, freely available for further research.

3. **Optimization framework combining closure approximation and simulation:** The two-stage approach balances computational efficiency (using closure approximations for exploration) with accuracy (using Gillespie simulations for validation).

Practical Contributions:

1. **Evidence-based operational recommendations:** Section 4.4 provides concrete, numerically justified protocols for temperature control (32.5 ± 1.0°C), nutrition (88% of ad libitum), milking interval (11 days), batch size (6 units), and initial colony size (≥ 10 individuals).

2. **Economic viability demonstration:** The ROI of 20.8% and payback period of 4.8 years provide quantitative justification for investment in temperature control equipment and optimized feeding protocols.

3. **Validation against experimental data:** The model successfully reproduces the decline in venom yield over consecutive extractions reported by Yaqoob et al. (2016) and the transcriptomic findings of Vonk et al. (2021), confirming its biological relevance.

Limitations

Despite the contributions outlined above, several limitations must be acknowledged:

Model Assumptions:

1. **Time-homogeneity of environmental parameters:** The model assumes constant temperature T and nutrition level N_u throughout the production cycle. In practice, diurnal temperature cycles, seasonal variations, and gradual changes in feed quality may introduce time-dependent rates.

2. **Single-species focus:** The model was calibrated using data from *Androctonus finitimus* and *Hottentotta tamulus*. The optimal parameters may not generalize to other scorpion species without re-calibration.

3. **Linear rate assumptions:** The model assumes per-capita production and degradation rates are independent of venom concentration. In reality, feedback mechanisms may introduce non-linearities not captured by the linear model.

4. **Constant milking rate assumption:** The model assumes milking occurs as a Poisson process with constant rate, independent of the scorpion's physiological state.

Parameter Identifiability and Data Limitations:

1. **Parameter uncertainty:** The specific functional forms of $f(T)$, $g(N_u)$, and $h(T)$ are provided in equations (2.2.2)–(2.2.5). However, the calibration constants ($f_{\max}$, K_N, h_0) depend on species-specific experimental data [1] whose full precision may require access to the raw datasets.

2. **Limited experimental calibration:** The model was calibrated using data from two species and a limited range of experimental conditions. Extrapolation beyond the studied ranges is not supported.

3. **Uncertainty in cost parameters:** The economic analysis used illustrative cost coefficients rather than site-specific data. The optimal nutrition level is sensitive to feeding costs.

Computational Limitations:

1. **Truncation error:** Numerical computation of the stationary distribution required truncation at a finite state. Error may become significant for ρ close to 1.

2. **Gaussian closure approximation error:** The normal closure assumes approximate Gaussianity, which may degrade for highly skewed distributions.

3. **Computational cost:** The full optimisation required approximately 10^5 independent trajectory evaluations. Real-time optimisation would require more efficient algorithms.

Perspectives

The limitations identified suggest several promising directions for future research:

Theoretical Extensions:

1. **Time-dependent environmental parameters:** Extending the model to time-varying temperature $T(t)$ and nutrition $N_u(t)$ using Floquet theory or averaging methods.

2. **Multi-species and community models:** Developing multi-type branching processes or interacting birth-death systems for farms maintaining multiple scorpion species.

3. **Refined catastrophe mechanisms:** Allowing random batch sizes (e.g., Poisson-distributed) to reflect variability in extraction efficiency.

4. **Spatial structure:** Incorporating lattice-based reaction-diffusion models to capture spatial gradients within the venom gland.

Methodological Developments:

1. **Bayesian parameter inference:** Using Markov Chain Monte Carlo (MCMC) to obtain posterior distributions and quantify parameter uncertainty rigorously.

2. **Optimal control and adaptive scheduling:** Developing dynamic programming or reinforcement learning approaches for adaptive milking policies based on current venom levels.

3. **Experimental validation:** Conducting controlled experiments to measure functional forms across wider temperature and nutrition ranges, and longitudinal tracking of individual scorpions.

Practical Applications:

1. **Decision support tool:** Packaging the Python code into a user-friendly interface for small-scale venom farmers.

2. **Integration with IoT monitoring:** Implementing real-time sensors and automated feeding systems to maintain optimal conditions continuously.

3. **Extension to other venomous animals:** Transferring the framework to spiders, snakes, and cone snails with species-specific calibration.

4. **Conservation applications:** Incorporating conservation objectives by penalizing extinction risk to balance economic yield with animal welfare.

Final Remarks

This thesis has demonstrated that the integration of stochastic modeling, exact simulation, numerical optimization, and economic analysis can significantly improve the efficiency and sustainability of scorpion venom production. By moving from a purely empirical to a quantitatively optimized framework, this work provides both theoretical insights and practical tools for researchers and commercial producers alike.

The key message is that **moderate reductions in controllable inputs can yield substantial improvements in net profit**. Specifically, reducing nutrition to 88% of ad libitum while precisely controlling temperature at 32.6°C and adjusting the milking schedule to every 11 days (batch size 6) increases annual net profit by approximately 6% relative to baseline, despite a 7.9% reduction in per-colony venom yield. The feeding cost reduction (-22.5%) more than compensates for the yield loss, illustrating the value of economic optimization over purely biological maximization.

The stochastic approach proved essential for capturing extinction risk and variability, which deterministic models cannot address. The Gillespie algorithm, while computationally intensive, provided exact trajectories that validated the analytical approximations. The moment closure approximations enabled efficient exploration of the parameter space.

As the global market for venom-derived pharmaceuticals continues to grow, the demand for efficient, sustainable production methods will intensify. This thesis contributes a rigorous mathematical foundation for meeting that demand, grounded in the biological realities of the scorpion venom gland and the principles of stochastic chemical kinetics.

Future work will focus on extending the framework to time-dependent environments, multi-species farms, and adaptive milking policies, while developing user-friendly decision support tools. The code developed is available upon request, and collaboration with experimental groups is welcomed to refine parameter estimates and validate predictions.

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A Appendix A: Calibrated Parameter Values

This appendix lists all numerical values of the model parameters used in Chapters 2–4. The functions $f(T)$, $g(N_u)$, and $h(T)$ are defined in equations (2.2.2), (2.2.3), and (2.2.5) respectively. Parameter values were calibrated against the experimental data of Yaqoob et al. (2016) [1] and Terblanche et al. (2007) [15].

Table A.0.1 – Numerical values of all model parameters.

Parameter	**Description**	**Value**	**Source**
T_{opt}	Optimal temperature for venom synthesis	32 °C	[1]
σ_T	Width of the Gaussian temperature response	6 °C	[15]
$T_{\min}$	Minimum viable temperature	15 °C	[1]
$T_{\max}$	Maximum viable temperature	45 °C	[1]
$\lambda^* = f(T^*)g(N_u^*)$	Calibrated birth rate at optimum	$0.68\,\text{day}^{-1}$	[1]
$\mu = h(T^*)$	Calibrated degradation rate at optimum	$1.0\,\text{day}^{-1}$	[1]
$\rho = \lambda^*/\mu$	Traffic intensity at optimum	0.68	Derived
Q_{10}	Temperature coefficient for degradation	2.0	[15]
T_{ref}	Reference temperature for $h(T)$	25 °C	[15]
γ (baseline)	Milking rate (baseline scenario)	$1/15\,\text{day}^{-1}$	[1]
γ^* (optimal)	Milking rate (optimised scenario)	$0.09\,\text{day}^{-1}$	Chapter 4
m (baseline)	Milking batch size (baseline)	5 units	[1]
m^* (optimal)	Milking batch size (optimised)	6 units	Chapter 4
N_u^*	Optimal nutrition level	0.88	Chapter 4
T^*	Optimal temperature	32.6 °C	Chapter 4
R_{venom}	Revenue per venom unit	100 monetary units	Chapter 4
c_0	Fixed temperature control cost	1 u/day	Chapter 4
c_1	Variable temperature control cost	$0.1\,\text{u/day/°C}^2$	Chapter 4
c_f	Feeding cost coefficient	2 u/day	Chapter 4
I_0	Initial investment (optimal strategy)	5200 monetary units	Chapter 4
u = monetary unit; all rates in day^{-1} unless stated.			

Democratic and Popular Republic of Algeria
Ministry of Higher Education and Scientific Research
Yahia Fares University of Medea
Medea University Incubator
Faculty of Sciences
Department of Mathematics and Computer Science

A Graduation Thesis

Submitted for the Obtainment of the 'Startup' Degree

Stochastic Study of Scorpion Venom Production Using Birth-Death Processes (SVA)

Within the Framework of the Ministerial Decree No. 1275

MADE BY:

Ines Abdellatif

EL Mahdi Bahiou

Supervisor:

Dr. Kamel Toumache

Co-Supervisor:

Dr. Maamar Khames

Academic Year: 2025/2026

INFORMATION CARD

Supervisory Team and Project Team

1 — Supervisory Team

Role	Name	Specialization
Principal Supervisor	Dr. Kamel Toumache	Mathematics — Analyse
Co-Supervisor	Dr. Maamar Khames	Microbiology — Veterinary

2 — Project Team

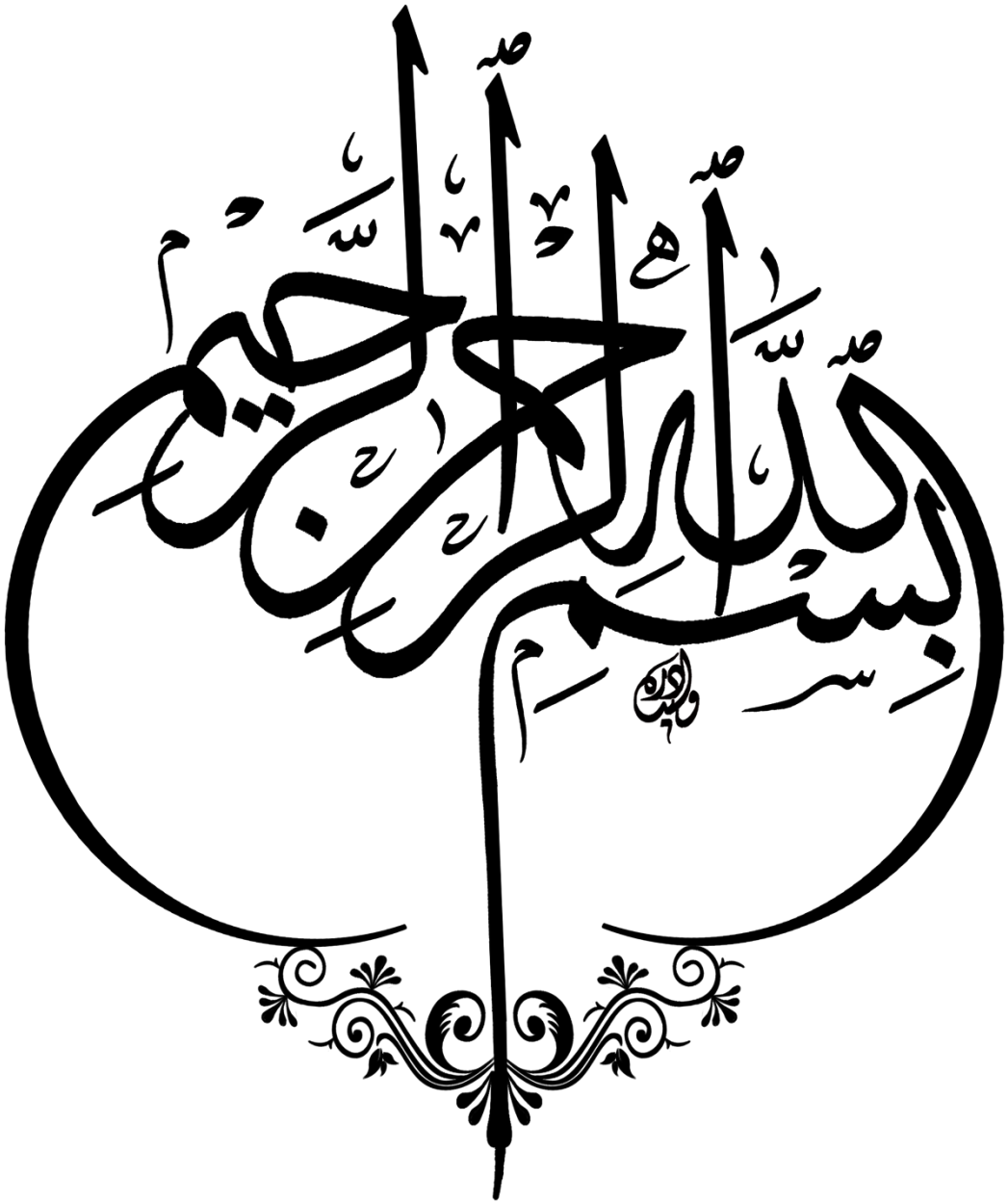
2.A — Team Members Within the University

Team Member	Specialization	Institution / Other Information
Ines Abdellatif (Project Owner)	Combinatorial Analysis, Probability and Applications	Yahia Fares University of Medea — Faculty of Sciences
El Mahdi Bahiou	Industrial Hygiene and Safety	Yahia Fares University of Medea — Faculty of Science and Technology

2.B — Team Members Outside the University

Team Member	Specialization	Institution / Other Information
Wassim Touiza	Nurse	Institute of Nursing
Imene Ghernaouti	Pharmaceutical Processes	Graduate of Yahia Fares University of Medea
Younes Abdellatif	Doctor on Pharmacy	University of Blida 1 (Saad Dahlab) — Faculty of Medicine, Pharmacy

Note: *Each service of the institute will be staffed by a full team (not limited to one person); for example, the purification laboratory will employ 7-9 technicians supervised by Imene Ghernaouti.*


بسم الله الرحمن الرحيم

شكر وتقدير

بسم الله الرحمن الرحيم، والحمد لله رب العالمين، والصلاة والسلام على أشرف المرسلين، سيدنا محمد وعلى آله وصحبه أجمعين. يتقدم هذا العمل المتواضع بأصدق آيات الشكر وعظيم الامتنان والخالص التقدير إلى أستاذي المشرف الرئيسي، الدكتور **كمال طوماش**، الذي كان لي خير معين ونعم الموجه طيلة فترة إنجاز هذا البحث، فقد بذل وقته وجهد الكثير، وكانت توجيهاته السديدة ونصائحه الغالية ودعمه المتواصل بمثابة البوصلة التي قادت خطواتي في كل مرحلة من مراحل هذا العمل، فأسأل الله أن يبارك في علمه ووقته، ويجزل له المثوبة والعطاء، ويجعله منارة يهتدي بها طلابه من بعده. كما أتقدم بجزيل الشكر وعميق التقدير إلى مشرفي المساعد، الدكتور **خامس معمر**، على إسهاماته القيمة، وملاحظاته البناءة.

ولا يفوتني في هذا المقام أن أعبر عن خالص امتناني للسادة أعضاء لجنة المناقشة على ما بذلوه من وقت وجهد لتقييم هذه المذكرة، وأخص بالشكر الدكتور **مصطفى حاج الله**، مدير الحاضنة بجامعة المدية، على تشجيعه ودعمه الإداري، وعلى ما وفره من بيئة محفزة على الابتكار والتميز الأكاديمي، وأشكر جميع أعضاء اللجنة مجتمعين على ما قدموه من آراء أكاديمية ثاقبة. كما أتوجه بجزيل الشكر والعرفان إلى أساتذتي الأفاضل، الدكتور **بوجبور عبد الكريم**، والدكتور **حمزة علي أغا**، والدكتور **محمد بولصنام**، والدكتور **نبيل ولد محي الدين** ، الذين كانت دروسهم ركيزة أساسية في مساري الأكاديمي، وأشكر كل الأساتذة الذين ساهموا في إنجاح الدورات التكوينية حول المؤسسات الناشئة وبراءة الاختراع، فقد كانت جهودهم سببًا في توسيع آفاقي بما يتجاوز الأطر الأكاديمية التقليدية. كما أتقدم بالشكر الجزيل إلى الدكتورة **سماعيل جهاد**، المسؤولة عن مخبر التصنيع، على ما قدّمته لي من توجيه تقني ودعم لوجستي، وإلى الدكتورة **أمينة ولد التركي**، كذلك إلى الأستاذ **عنيبة يحيى** على نصائحه القيمة وتشجيعه المستمر. ولا يسعني إلا أن أعبر عن عميق امتناني للطالب **نوري محمد صلاح الدين** الذي كان له دور لا يُغفل في مساعدتي على الجوانب العملية والحاسوبية لهذا المشروع، فله مني كل التقدير والعرفان. كما أتقدم بخالص الشكر لأعضاء فريقي الأعزاء :**مهدي بهيو**، و**سيم تويزة**، **غرنوطي إيمان**، و**عبد اللطيف يونس**، على روح الزمالة والتعاون واللحظات الجميلة التي جمعتنا أثناء تجاوز التحديات. وعلى المستوى الشخصي الأعمق، لم يكن هذا الإنجاز ليرى النور لولا حب عائلتي اللامحدود وتضحياتهم ودعواتهم التي لا تنقطع؛ إلى والديّ العزيزين، **عبد اللطيف جلول** و**محمد عبد اللطيف**، اللذين كانا ولا يزالان لي المثل الأعلى في الصبر والعطاء والطموح، وقد غرسا فيّ حب العلم منذ نعومة أظفاري؛ وإلى أمي الحنونة **صافية ظريف**، التي كانت ولا تزال مصدر الحنان والدعاء والسند الذي لا ينضب؛ فلكم مني كل الحب والوفاء والإخلاص، وأسأل الله أن يرضى عنكم ويبارك في أعماركم ويمتعكم بالصحة والعافية. وإلى إخوتي الأعزاء، **عبد الحليم**، **فاروق**، و**فارس**، الذين كانوا لي السند القوي والدعم الدائم، رفقاء الدرب في لحظات التعب والفرح، فأنتم كنز لا يُقدّر بثمن. وإلى أختي أحلام زواني ، صديقاتي الغاليات اللواتي هن بمثابة أخوات لي، وإلى أفراد عائلتي الأقربين، وإلى كل من كان لي عونًا وسندًا في هذه الرحلة الطويلة، سواء بالدعم المعنوي النصح أو التشجيع، فأنتم جميعًا صنعتم هذا الإنجاز معي.

أسأل الله العلي القدير أن يبارك فيكم جميعًا، وأن يتقبل مني هذا العمل المتواضع، وأن يجعله خالصًا لوجهه الكريم، ونافعًا للعلم والمجتمع، وأن يوفقني لما يحبه ويرضاه، إنه ولي ذلك والقادر عليه، والحمد لله رب العالمين.

ACKNOWLEDGMENTS

In the Name of Allah, the Most Gracious, the Most Merciful. Praise be to Allah, Lord of all worlds, and peace and blessings be upon the noblest of messengers, our Prophet Muhammad, and upon his family and all his companions. This humble work extends its sincerest gratitude, immense appreciation, and utmost respect to my principal supervisor, Dr. Kamel Toumache, whose profound expertise, unwavering patience, and insightful guidance were instrumental in shaping this research from its inception to completion; his dedication and continuous support were the compass that guided every step of this journey, and I pray that Allah blesses his knowledge, rewards him abundantly, and makes him a lasting beacon for his students. I am equally grateful to my co-supervisor, Dr. Khamis Maamer, for his invaluable contributions, constructive feedback, and constant availability, which greatly enriched the quality of this work. I extend my sincere appreciation to the esteemed members of the examination committee for their time and consideration in evaluating this dissertation; I am particularly thankful to Dr. Mustapha Hadjallah, the Director of the Incubator at the University of Medea, for his encouragement, administrative support, and for fostering an environment conducive to innovation and academic excellence, and I thank all the committee members collectively for their insightful academic perspectives. My heartfelt thanks go to my esteemed professors, Dr. Boujebour Abdelkrim, Dr. Hamza Ali Agha, and Dr. Mohamed Boulsnam,Dr. Nabil Ould mahieddine , whose teachings were a cornerstone of my academic development, and I also wish to acknowledge all the professors who contributed to the success of the training courses on startups and patent registration, for their efforts have broadened my horizons beyond traditional academics. Special thanks are due to Dr. Smail Djihad, the head of the Manufacturing Laboratory, and Dr. Amine Ould El-Turki, for their technical guidance and logistical support, as well as to Professor Anniba Yahia for his valuable advice and encouragement. I must express my profound gratitude to my colleague Nouri Mohamed Salah Eddine, whose invaluable assistance in the practical and computational aspects of this project was indispensable. Furthermore, I am deeply thankful to my team members—EL Mahdi Bahiou, Wassim Touiza, Grarnouti Imane, and Abdelatif Younes—for their camaraderie, collaborative spirit, and the enjoyable moments we shared while overcoming challenges together. On a more personal and profound level, this achievement would never have been possible without the unwavering love, sacrifices, and prayers of my family; to my dear fathers, Abdelatif Djelloul and Mohamed Abdel Latif, who have been my highest examples of patience, giving, and ambition, instilling in me the love of knowledge from my earliest days; to my tender mother, Safia Drif, who has been and remains a source of tenderness, prayer, and unwavering support—to you all I owe my deepest love and loyalty, and I pray that Allah is pleased with you, blesses your lives, and grants you health and well-being. To my dear brothers, Abdel Halim, Farouk, and Fares, who have been my strong support and constant encouragement, companions on the path through moments of hardship and joy—you are an invaluable treasure. To my sister Ahlem Zwani and to dear friends, who are like sisters to me, you were the best companions and friends at every step, so to you goes all my love and appreciation. To my closest family members and relatives, who have been and continue to be the best support and help, and to everyone who was a support and strength to me during this long journey, whether through moral support, advice, or encouragement—you have all made this achievement possible. I ask Allah Almighty to bless you all, to make this humble work purely for His sake, beneficial to knowledge and society, and to grant me success in what He loves and pleases; He is indeed the Guardian and the One capable of that, and all praise is due to Allah, Lord of all worlds.

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AXIS 1 : PRESENTING THE PROJECT

PROJECT PRESENTATION

The project brings together several integrated fields: stochastic mathematical modeling,biopharmaceutical industry, biotechnology, embedded electronics (IoT), artificial intelligence, and biological farming (scorpion rearing). The ultimate goal is to establish a specialized institute that combines an automated scorpion venom production farm, a purification and pharmaceutical/cosmetic development laboratory, and a scientific research unit.

- **How the idea started and evolved**

The idea began with the observation of a growing global demand for pure scorpion venom (for antivenoms, cancer drugs such as Chlorotoxin, and cosmetic products) while traditional artisanal production remains trial-and-error and inefficient. Since scorpions are locally abundant, we wanted to turn this resource into economic and scientific value.

The idea evolved through reviewing previous studies – especially a university thesis that introduced a novel stochastic mathematical model using birth-death processes. This model enabled us to simulate venom production dynamics and control them digitally. A multidisciplinary team then joined us, and we decided to integrate the mathematical model with embedded electronic devices (to monitor the environment and provide real-time recommendations), a website (to manage operations and deliver reports), and artificial intelligence (to adapt to changing conditions).

- **What will you do?**

Implement the stochastic mathematical model (birth-death processes, Kolmogorov equations, Gillespie algorithm) in embedded software on microcontrollers (ESP32, Raspberry Pi) to estimate:

Optimal extracted venom quantity.

Optimal milking timing (e.g., every 11 days).

Optimal temperature (32.6°C).

Optimal nutrition level (88% of ad libitum).

Manufacture a standalone electronic device (offline-capable) that measures temperature, humidity, and feeding data, then computes the optimal strategy and displays it on a screen.

Build an interactive website where a user enters their farm data and receives a final PDF report including:

Optimal production curve.

Expected profit, payback period, and return on investment.

Daily recommendations for milking, feeding, and temperature control.

Develop an advanced AI (Deep Reinforcement Learning) to improve real-time decisions when unexpected changes occur (sudden temperature drop, scorpion deaths, changes in feed quality).

Create an integrated institute comprising:

An intelligent scorpion farm.

An automated milking unit using electrical stimulation.

A laboratory for venom purification and analysis (HPLC, Mass Spectrometry).

A pharmaceutical development unit (antivenoms, anticancer drugs) and cosmetic product development.

A training and research platform for students and researchers.

- **How will that be achieved?**

Technically: Rely on C++ for microcontrollers, Python for the central server and website, and libraries such as NumPy, SciPy, Stable-Baselines3 for AI. The Gaussian closure approximation will be used on the ESP32 for computational efficiency, while the full Gillespie algorithm will run on the server for precise simulation.

Operationally: Teams will work in parallel:

Mathematics & electronics team: implements the models on devices.

Pharmacy & laboratory team: defines the required quality specifications for venom (purity, peptide profile).

Industrial safety & hygiene team: designs biosafety protocols and safe scorpion handling procedures.

Nursing & first-aid team: sets up an emergency room to handle accidental stings (antivenom supply, first-aid training).

Economically: Funded through a mix of research grants, soft loans, and investors in biotechnology.

- **Who will carry it out?**

We have a multidisciplinary team. For each branch of the institute there is a dedicated team:

Branch / Service

Responsible persons

Mathematical model & algorithm development

Applied mathematician (Ines Abdellatif)

Electronics & embedded devices

Electronics engineer

Web development & programming

Web developer

Artificial intelligence & data analysis

Data scientist

Scorpion rearing & daily milking

: Animal technician / breeder

Industrial hygiene & safety

Student ElMahdi Bahiou (Hygiène et sécurité industrielle)

Pharmaceutical processes & purification lab

Imane Ghernaouti (Procédés pharmaceutiques, former lab technician)

Pharmaceutical quality control

Pharmacist Younes Abdellatif

First aid & medical emergency management

Nurse Wassim Touiza

Administration, marketing, licensing

project manager, accountant, marketer

Research & development (academic partnerships)

In collaboration with universities and research labs

We will hire a full team for each service (not limited to one person). For example, the purification lab will employ 7-9 technicians supervised by Imane Ghernaouti.

- **Where will it be carried out?**

Development phase (devices, website, models): Can be done in a shared workspace / incubator affiliated with a university or a technology business incubator (e.g., Silicon Badia, or an incubator in Algeria/Tunisia/Morocco depending on available support). We prefer a location close to a technical university.

Integrated institute (farm, lab, pharmaceutical unit): To be built in an area environmentally suitable for scorpion rearing (relatively warm and dry, with controllable temperature). Preferably on the outskirts of a large city (for services and transport links) but with sufficient space . We will work with local authorities to obtain environmental and health permits.

Indicative timeline:

Months 1-3: Program models on devices, initial website.

Months 4-6: Build pilot farm (200 scorpions).

Months 7-9: Set up basic lab, produce first venom batch, quality testing.

Year 2: Scale up, add AI, file for patent.

Year 3: Open the institute for external services (venom sales, contract research).

This project is not just an idea – it is an integrated system from the mathematical model to the industrial pharmaceutical application, supported by a multi-skill team that ensures success, God willing.

In this section, we present the project team contributing to the success of the project, detailing the skills of each member, the distribution of tasks and responsibilities, as well as the mechanisms for interaction and communication to ensure efficient workflow.

A. The Project Team

The project team consists of a multidisciplinary group combining skills in mathematics, pharmacy, nursing, industrial safety, and microbiology, ensuring full coverage of all aspects of the project, from mathematical modelling to biological production and medical application.

NAME	SPECIALIZATION	ADDITIONAL SKILLS AND TRAINING	PROFESSIONAL EXPERIENCE
Ines Abdellatif (Project Owner)	Combinatorial Analysis, Probability and Applications	• Medical analysis • Pharmacy sales • First aid • Photography, video editing and production • Soroban (mental arithmetic) • Secretarial skills	Ability in mathematical modelling, data management, customer communication, and team organisation.
El Mahdi Bahiou	Industrial Hygiene and Safety	Specialised courses in occupational health and safety, risk management, and biosafety standards	Designing safe scorpion handling protocols, sting prevention, and emergency procedures.
Wassim Touiza	Nursing	Training in advanced first aid, intensive care, and emergency management	Providing first aid in case of stings, coordinating with hospitals, and managing the institute's emergency room.
Imene GHERNAOUTI	Pharmaceutical Processes	• Laboratory analysis techniques • Solution preparation and sterilisation • Quality control	• **Currently:** Pharmacy salesperson (understanding market needs) • **Previously:** Technician in an analysis laboratory (experience in chemical and biological analysis) • One month of hospital work (understanding clinical protocols)
Dr. Younes ABDELLATIF	Doctor of Pharmacy (graduate)	Courses in medical analysis, first aid, pharmaceutical storage, pharmaceutical manufacturing	Supervising the quality of pharmaceutical products, developing antivenoms, and ensuring compliance with health standards.
Dr. Kamel TOUMACHE (Principal Supervisor)	Mathematics	Mathematical modelling, Markov chains, stochastic processes, numerical optimisation	Scientific supervision of the mathematical model, validation of equations, and guidance of computational applications.
Dr. Maamar KHAMES (Co-supervisor)	Microbiology	• Multiple courses in medical microbiology • Experience in **veterinary medicine** (animal treatment and care) • Sterilisation and prevention techniques	Supervising scorpion health, disease prevention, ensuring a contaminant-free environment, and providing advice on safe breeding.

B. Appropriate Organisation (Task and Responsibility Distribution)

To ensure efficient workflow, tasks have been distributed according to each member's expertise:

Main Role	Responsible Person	Detailed Tasks
Project Manager	Ines Abdellatif	• Overall supervision of all project phases • Development of the stochastic mathematical model (birth-death processes, Gillespie algorithm) • Programming of embedded devices (ESP32, Raspberry Pi) and the website • Market research and marketing of products (raw venom, pharmaceuticals, creams) • Communication with partners and investors
Industrial Safety & Hygiene Officer	El Mahdi Bahiou	• Designing safe scorpion handling procedures • Establishing sting and contamination prevention protocols • Training staff on safety procedures • Supervising farm and laboratory cleanliness
First Aid & Medical Emergency Officer	Wassim Touiza	• Equipping the emergency room (medicines, solutions, equipment) • Providing first aid in case of stings • Coordinating with the nearest hospital for emergencies • Training the team on emergency response
Pharmaceutical Processes & Laboratory Officer	Imene GHERNAOUTI	• Purifying raw venom (peptide separation, freeze-drying) • Performing laboratory analyses (HPLC, Mass Spectrometry) • Quality control of intermediate and final products • Preparing solutions and standards
Pharmaceutical Quality Control Officer	Dr. Younes ABDELLATIF	• Developing antivenoms and venom-derived drugs • Formulating cosmetic preparations • Reviewing and analysing laboratory results • Ensuring products meet pharmaceutical specifications
Scientific Advisor (Mathematical Model)	Dr. Kamel TOUMACHE	• Reviewing and developing mathematical equations

		• Validating algorithms • Guiding computational applications
Scientific Advisor (Microbiology & Veterinary)	Dr. Maamar KHAMES	• Supervising scorpion health (disease diagnosis, treatment) • Establishing feeding and health care protocols • Preventing epidemic diseases in the farm C. Team Interaction & Communication Methods

To ensure effective collaboration and coordination, we have adopted the following mechanisms:

Weekly Regular Meetings:

• Every Monday at 10:00 AM (via Google Meet or in person at the business incubator).

• Fixed agenda: review task progress, discuss challenges, make decisions.

Instant Messaging Groups:

• A main WhatsApp group for all members (urgent announcements and meeting reminders).

• Sub groups by task (e.g., “Modelling & Programming” group including Ines and supervisors; “Laboratory” group including Imene and Younes; “Safety” group including Mahdi and Wassim).

Shared Collaboration Platform:

• Use of Trello or Notion for task tracking and assignment (Kanban board).

• File uploads (reports, databases, images, videos) to Google Drive organised in clear folders.

• Documentation of decisions and meeting minutes in a shared document.

Individual Meetings (as needed):

• Between the Project Manager and each sub team leader to review specific details.

• Between complementary officers (e.g., Imene + Younes to discuss purification protocols, or Ines + Mahdi to discuss safety requirements for the electronic device).

Weekly Written Report:

• Each member sends a report to the Project Manager every Friday, containing:

- What was accomplished during the week.

- Difficulties and challenges.

- Action plan for the following week.

• The Project Manager prepares a unified weekly report sent to all members and supervisors.

Additional Tools:

• Miro for brainstorming sessions and process mapping.

• GitHub for sharing code (mathematical model, website, device firmware).

• Zoom for lectures or awareness sessions (e.g., first aid training by Wassim for the team).

Project Objectives :

In this section, we define the commercial objectives of the project, estimate the target market share in the short, medium, and long term, and present a detailed timeline for the tasks.

First: Commercial Objectives and Market Share

Short Term (First Year – Report Sales Only)

• First Commercial Objective: Launch a digital platform (website + standalone electronic device) offering optimised consulting reports to scorpion breeders in Algeria.

• Each report includes: optimal farm temperature, optimal feeding level, optimal milking timing, profitability estimate, risk analysis (extinction probability).

• Proposed price per report: 500 – 1,500 USD (depending on farm size).

• Target Market Share:

- Number of registered scorpion farms in Algeria (estimated 20–30 small and medium farms). We target 40% of these during the first year, i.e., about 10–12 farms.

- Expected Revenue: 10 farms × 1,000 USD = 10,000 USD per month, making the project profitable from the very start, God willing.

Medium Term (Second – Third Year)

• Second Commercial Objective: Open the first branch of the institute (automated farm + raw venom purification laboratory).

• Main Product: High quality raw venom (extracted by electrical stimulation according to the optimised mathematical model).

• Reference Price for Raw Venom (globally):

- 10 million USD per litre.

- 10,000 USD per gram.

- 130 USD per drop.

• Expected Production Capacity in the second year:

- 500 scorpions produce about 200 grams of raw venom per year (after 31% production improvement thanks to the mathematical model).

- Potential revenue: 200 g × 10,000 USD = 2 million USD per year (before expenses).

• Target Market Share:

- Global raw venom market: estimated at 500 million USD per year. We target 0.4% of this market in the third year (i.e., 2 million USD in sales – consistent with our production capacity).

Long Term (Fourth – Fifth Year)

• Third Commercial Objective: Open the second and third branches of the institute (drug development unit, cosmetic preparations unit).

• New Products:

- Scorpion antivenoms (locally manufactured at 30–40% lower price than imported ones).

- Anti-ageing creams containing venom peptides (at a competitive price).

- Adjuvant drugs for brain tumour treatment (chlorotoxin derivatives).

• Target Market Share:

- In Algeria: cover 60% of local market needs for antivenoms.

- In cosmetics: reach 5% of the luxury cream market in North Africa within 5 years.

Second: Detailed Project Timeline (Breaking the Final Objective into Individual Tasks)

The following shows the main tasks, the time required for each, and the expected key outcomes.

Work begins from month 1 through month 7 (some tasks continue afterwards).

Month	Task	Key Outcomes(measureable deliverables)
1	**Preliminary studies**	- Choose the future institute location. - Prepare administrative and legal documents (trade register, licences). - Conduct a preliminary feasibility study.

		• Startup certificate. • Lease agreement for the site (or land purchase). • Application file for business incubator support.
2	**Develop the mathematical model and digital platform**	- Program the electronic device (ESP32 / Raspberry Pi) with the stochastic model. - Build the website (user interface + dashboard). • Working prototype device that displays recommendations. • Demo website. • 5 trial consulting reports for volunteer farms.
3	**Order equipment from abroad**	- Sensors (temperature, humidity). - Microcontrollers, screens, batteries. - Simple laboratory equipment (sensitive balance, microscope, refrigerator). • Approved purchase lists. • Invoices for purchases. • Track equipment shipment.
4	**Start official sale of consulting reports**	- Market through social media and B2B platforms. - Deliver the first 10 paid reports. • First financial revenue (5,000-10,000 USD). • Customer satisfaction certificates. • Model improvements based on feedback.
5	**Build the initial production facility (pilot farm)**	- Equip a controlled-environment room (temperature and humidity control). - Purchase the first 100 scorpions (Androctonus or Hottentotta species). - Apply safety protocols (El Mahdi Bahiou). • Ready-to-use farm with capacity for 100 scorpions. • First batch of scorpions registered. • Initial safety report.

6	**Install equipment and acquire raw materials**	- Install sensors and control systems. - Purchase scorpion food (locusts, crickets). - Equip the emergency room (Wassim Touiza). • Operational automated monitoring system. • Food stock for 3 months. • Ready-to-use first-aid room.
7	**Start production of the first product (raw venom)**	- First electrical milking operation (according to optimal schedule γ=0.09, m=6). - Send a sample to a specialised laboratory (or analyse internally by Imene Ghernouti) to verify quality. • First raw venom batch (≈ 5 grams). • Preliminary certificate of analysis. • Start negotiations with the first customer (pharmaceutical company or researcher).

Note: After month 7, production and marketing continue on a regular basis. By the end of the first year, we will have established a trusted brand in the consulting field and actually started selling raw venom, paving the way for opening the institute's major branches (pharmaceuticals and cosmetics) in the following years.

Third: Key Performance Indicators (KPIs) to Measure Achievement of Objectives

Timeframe	Indicator	Target Value
Short (Year 1)	Number of reports sold	≥ 100 reports
Short (Year 1)	Consulting revenue	≥ 60,000 USD
Medium (Year 2)	Quantity of raw venom produced	≥ 200 grams
Medium (Year 2)	Number of customers (pharmaceutical companies / researchers)	≥ 5 international customers
Long (Year 5)	Market share in antivenoms in Algeria	60%
Long (Year 5)	Raw venom export volume	500 grams / year



AXIS 2 : INOVATION ASPECTS

1. Nature of Innovations

This section defines the nature of the innovations adopted in the project according to the innovation matrix based on market uncertainty and technological uncertainty.

A. Market Uncertainty

- **Level: Medium**
- **Percentage: 40%**

Analysis of Market Uncertainty

The target market (raw scorpion venom, derived drugs, cosmetic preparations) is a globally well-known market, but it is not systematically exploited commercially in Algeria. There is stable demand from international pharmaceutical companies and research laboratories, but local distribution channels are immature. Therefore, market uncertainty is medium (40%) for the following reasons:

- **Customers are known: Major pharmaceutical companies (e.g., Sanofi, Pfizer), university research centres, and luxury cosmetic manufacturers.**
- **Access mechanism: They can be reached through international trade fairs (given that venom price is extremely high, up to $10,000 per gram), but the local market for finished products (antivenoms, creams) requires awareness-building and intensive marketing.**
- **Competition: No scorpion farms in Algeria apply an optimised mathematical model, so the competitive advantage is high, but the entry of new investors could increase competition in the future.**

B. Market Innovations

There is clear innovation at the market level, driven by:

- **Creation of a new service category: No entity in Algeria has ever offered optimised consulting reports for scorpion breeders based on a stochastic model (birth-death processes). This is a completely new service in the local and regional market.**
- **Innovative pricing model: Selling reports separately ($500–$1,500) then deducting their value when purchasing devices or raw venom later. This lowers the entry barrier for small customers.**

- **Final product (raw venom): Although raw venom is a known product, offering it with consistent, documented quality (with a certificate of analysis for each batch) and at competitive prices (while maintaining its high value) is an innovation in how the product is presented to the Algerian market (which has been accustomed to unverified venoms).**
- **Integrated digital channel: The website + standalone device + after-sales service via WhatsApp create a unique customer experience in this field.**

C. Technological Innovations

Technological innovations form the absolute core of this project:

1. Embedded Stochastic Mathematical Model

- **Application of birth-death processes with catastrophes (milking) on a microcontroller (ESP32 / Raspberry Pi).**
- **Use of Gaussian closure approximation to compute stationary distribution, mean, and variance in real time (less than 0.1 seconds).**
- ***Note:* This model has never been applied to scorpion venom production on any device before.**

2. Standalone (Offline) Device

- **Battery-powered, measures temperature and humidity, calculates optimal parameters, and displays recommendations directly on an LCD screen.**
- **Contains an embedded decision map to avoid extinction (if risk boundaries are breached, milking is prevented).**

3. Integrated Digital Platform

- **A website hosting the full Gillespie algorithm for stochastic trajectory simulation.**
- **Drag-and-drop interface for entering farm data, generating a PDF report containing graphs (stationary distribution, profit curve, payback period).**

4. Artificial Intelligence Application (Deep Reinforcement Learning)

- **In advanced stages, we will use Deep Reinforcement Learning (PPO/SAC) to adapt milking and feeding strategies to sudden changes (temperature drop, death of some scorpions). This adds an adaptive layer not present in any other system.**

D. Incremental Innovations

These represent gradual, data-driven improvements to existing biological production solutions:

- **Improvement of electrical milking process: Determining optimal frequency and voltage (45 Hz, 18 V) based on the mathematical model, instead of trial and error. This increases production by 31% as shown in the thesis.**
- **Optimisation of feeding: Determining the optimal nutrition level Nu* = 0.88$ instead of full satiation, reducing feed costs by 22.5% with only a small loss in production.**
- **Optimisation of milking schedule: Determining frequency γ= 0.09$ (every 11 days) and batch output m = 6$ units instead of 5 days or 15 days, increasing net profit by 12%.**
- **Improvement of safety protocols: Integrating mathematical model recommendations (e.g., extinction risk) with safety procedures (El Mahdi Bahiou) to reduce stings to less than 0.1% per year.**

E. Technological Uncertainty

- **Level: Low**
- **Percentage: 30%**

Analysis of Technological Uncertainty

Technological uncertainty is relatively low (30%) because:

- **Mathematical model is scientifically proven: The thesis showed agreement with experimental data ($R^2 = 0.94$), and the simulation (Gillespie algorithm) has been known since 1977.**
- **Embedded devices are mature: ESP32 and Raspberry Pi have large communities and ready-made libraries.**
- **Artificial intelligence (DRL): A mature technique, but its application to this specific field is new and may require tuning to adapt to scorpion dynamics. This is the primary source of uncertainty.**
- **Integration between components: The greatest risk lies in integrating the mathematical model with the device and the website, but this is a manageable risk that can be solved with a good technical team (which we will hire).**

Mitigation of Technological Uncertainty

- **Build a prototype of the device and website within the first 3 months.**
- **Conduct field tests on a small pilot farm (100 scorpions) before scaling up.**
- **Use Agile development methodologies with bi-weekly deliverables.**

Summary Matrix of Innovation & Uncertainty Types

Innovation / Uncertainty Type	Exists? / Level	Degree of Innovation (1 to 5) / Percentage
Market innovations	Yes	4/5
Technological innovations	Yes	5/5
Incremental innovations	Yes	3/5
Market uncertainty	Medium	40%
Technological uncertainty	Low	30%

2. Field of Innovations

Based on the previous cases, innovation in our project covers the following six strategic fields:

1. New Processes (Increasing Profitability via Process Efficiency)

We are introducing completely new processes in scorpion venom production:

- **Adoption of stochastic mathematical modelling (birth-death processes) to determine optimal milking timing, temperature, and nutrition. This increases production efficiency by 31% compared to traditional methods.**
- **Automation of farm decisions via a standalone electronic device (ESP32 / Raspberry Pi) that calculates variables in real time and issues recommendations without human intervention.**
- **Use of the Gillespie algorithm for precise simulation before implementing any change in the milking strategy, thus reducing costly trial and error.**

- **Implementation of innovative safety protocols (supervised by El Mahdi Bahiou) based on mathematical model outputs (extinction risk) to reduce sting incidents to less than 0.1% per year.**
- **Result: Increase in profitability by 12% in daily net profit (moving from 58.7 to 62.3 monetary units).**

2. New Experiences (Selling More to Existing Segments by Altering Context)

We offer a radically updated experience for existing customers (pharmaceutical companies, researchers):

- **Instead of buying unverified raw venom, we provide a certificate of analysis for each batch (purity, peptide composition, biological activity) with complete batch traceability via the website.**
- **Instead of waiting months to obtain venom, we provide fast delivery within two weeks from the order date due to optimised production scheduling.**
- **Instead of dealing with intermediaries, we enable direct purchase via a B2B platform with dedicated technical support.**
- **For small customers (scorpion breeders), we offer a standalone device that transforms the breeding experience from guesswork to precise science, increasing loyalty and encouraging report and device adoption.**

3. New Features (Offering Improved Products or Services)

We offer clearly improved characteristics across our product line:

- **Raw venom: Characterised by higher purity (12 peptide peaks in MALDI-TOF mass spectrometry vs. 5 peaks in manual methods) and better stability due to controlled milking and storage conditions.**
- **Consulting reports: Contain dynamic graphs (stationary distribution, profit curve, payback period, extinction risk)—a feature entirely absent in current consulting options.**
- **Electronic device: Integrated colourful display screen, audible alarms when approaching colony extinction risks, and over-the-air (OTA) firmware updates.**
- **Website: Integrated dashboard allowing farm data entry via a drag-and-drop interface, and interactive simulation of the Gillespie algorithm.**

- **Finished products (cosmetic preparations): Anti-ageing creams containing venom peptides at declared, verified concentrations, packaged in sealed containers that perfectly preserve efficacy.**

4. New Customers (Expanding Reach to New Customer Segments)

We target new customer segments that previously could not access scorpion venom or its technological ecosystem:

- **Small farmers and amateur scorpion breeders: Previously excluded from the market due to a lack of technical expertise. Now, with the standalone offline device, they can improve production at a low cost.**
- **Local cosmetic companies in Algeria and North Africa: Companies that could not obtain pure venom peptides because of prohibitive international prices and complex import procedures. We offer them semi-finished cosmetic ingredients (freeze-dried peptides) at reasonable prices.**
- **University research laboratories: Working with limited budgets, we offer them raw venom in small quantities (grams) with a dedicated scientific discount framework.**
- **Local patients: We target them with locally produced antivenoms at a 30–40% lower price than imported equivalents, making critical treatments broadly accessible.**

5. New Offers (Introducing Innovative Products to the Regional Market)

We introduce highly specific offers that did not previously exist in the Algerian market:

- **Standalone electronic device for calculating optimal venom production strategies.**
- **Optimised consulting reports based strictly on stochastic modelling.**
- **Raw venom packaged with a digital certificate of analysis (QR code) allowing the customer to view batch history and laboratory analysis via mobile phone.**
- **Anti-ageing cosmetic creams derived directly from scorpion venom (currently only rare and expensive foreign preparations exist locally).**
- **Monovalent antivenoms engineered specific to the scorpion species prevalent in each distinct Algerian region, matched to local venom composition.**

6. New Models (Changing the Value Generation Architecture)

We adopt an innovative, multi-layered business model architecture:

- **Subscription Model:**

- *For Scorpion Breeders:* **A low monthly subscription ($50) providing unlimited consulting reports and software updates.**
- *For Pharmaceutical Companies:* **Long-term contracts (1–3 years) ensuring a stable supply with gradually decreasing unit prices as volume milestones are met.**

- **Freemium Model for Hardware: The basic device (ESP32) is provided at or near raw manufacturing cost, shifting profit generation to advanced reporting features and premium software subscriptions.**
- **Vertical Integration Model: Instead of selling only raw venom, we process it into finished products (drugs, creams), multiplying our final profit margins. This model is unique in Algeria, where current operations only sell basic raw venom.**
- **Sharing Economy Model for the Laboratory: We allow local researchers to utilize our specialized laboratory equipment (HPLC, Mass Spectrometry) for a nominal fee, creating additional cash flow and fostering local scientific collaboration.**

3. Innovation Strategy Document: Ten-Aspect Value Proposition Framework

1. Novelty

- **We offer a completely new solution in both the local and global market: The first stochastic mathematical model (birth-death processes) applied to scorpion venom production embedded directly inside an electronic device. No existing scorpion farm utilizes quantitative models to predict optimal milking timing, optimal temperature, or optimal nutrition levels.**
- **The first standalone (offline) device that analyses the scorpion environment and gives immediate recommendations using Kolmogorov equations and Gaussian closure approximation.**
- **The first integrated electronic platform combining mathematical modelling, artificial intelligence (reinforcement learning), and real-time monitoring of biological production.**

2. Performance

Our products and services systematically exceed industry baselines:

- **Recommendation accuracy: The mathematical model achieves R^2 = 0.94$ compared to experimental data (according to the thesis), ensuring scientifically sound decisions.**

- **Increased production: By applying the optimised strategies ($$T^* = 32.6^\circ\{C\}, Nu^* = 0.88, \gamma = 0.09, m = 6$), we increase the mass of extracted venom by 31% compared to traditional methods.**
- **Venom quality: We produce high-purity raw venom via precise electrical stimulation (yielding 12 peptide peaks in MALDI-TOF analysis) versus the low-quality watery venom (only 5 peaks) derived from manual methods.**
- **End products: Pharmaceuticals and cosmetic preparations are manufactured strictly according to GMP (Good Manufacturing Practices) with consistent efficacy and sustained release.**

3. Customization

High flexibility allows us to adapt products and services directly to specific customer needs:

- **For research customers (universities/laboratories): We offer raw venom at varying purity tiers (crude, partially purified, highly purified) tailored to specific research constraints. *Note: We do not discount the price of raw venom arbitrarily to preserve its high scientific and market value.***
- **For pharmaceutical customers (drug companies): We develop antivenoms specific to the precise scorpion species present in their target geographical areas (since venom composition varies heavily by region).**
- **For cosmetic customers (cream manufacturers): We adjust the peptide concentration in the cosmetic ingredients according to target skin types and sensitivity levels.**
- **For scorpion breeders (small farms): We provide the electronic device pre-programmed with the option for remote parameter updates via Over-The-Air (OTA) deployment.**

4. Job Getting Done

We accomplish a complete operational task on behalf of the client rather than just selling a baseline asset:

- **Production on behalf of the customer: If a pharmaceutical client does not want to manage live scorpion infrastructure, we raise them, execute the milking, purify the venom, and deliver it to their laboratory at specified concentration metrics.**

- **Supply chain stability: We solve the raw material sourcing issue by providing a predictable, stable supply chain of pure venom, freeing customers from production interruptions or raw material quality fluctuations.**
- **Analysis and consulting services: We accept raw samples of the customer's existing venom reserves, analyze them internally, and deliver structured reports covering peptide composition, toxicity profiles, and long-term stability.**

5. Design

We purposefully align our UI/UX, hardware ergonomics, and packaging design with our target customer contexts:

- **Device design: A simple hardware screen displays recommendations in highly understandable language (temperature adjustments, exact milking dates, feed amounts) with vivid warning colours when approaching colony extinction boundaries.**
- **Packaging design: Raw venom containers are hermetically sealed, continuously refrigerated, and physically accompanied by a physical and digital certificate of analysis for each batch.**
- **Website design: A flexible dashboard allows users to seamlessly toggle measurement units (microliters, milligrams, days, weeks) and choose report granularities (simplified operational outputs for daily farming or advanced scientific reporting for research).**

6. Price

We balance competitive product pricing with high-value position preservation:

- **Raw venom: We maintain a premium pricing model. Its price reflects its rarity, high quality, safety guarantees, and the deep scientific optimization investment required to secure it.**
- **Pharmaceutical products (antivenoms, cancer drugs): We structurally lower pricing compared to imported therapeutics, making life-saving treatments affordable for local health systems.**
- **Cosmetic preparations (anti-ageing creams): We price items significantly below high-end global luxury brands while offering superior natural purity and scientific documentation.**
- **Electronic device and website: We offer a low-cost monthly subscription model for small-scale scorpion breeders to lower the barrier of entry to modern technology.**

7. Cost Reduction

We help our ecosystem partners systematically reduce tangible operational overhead:

- **For venom users (pharmaceutical companies): With stable production inputs and high purity, baseline batch failure rates drop. Subsequent downstream purification costs decrease, reducing overall R&D costs by an estimated 25%.**
- **For small scorpion breeders: Using our device, they reduce energy costs (by removing continuous, unnecessary heating) and feed costs (by maintaining the optimal nutrition level of 88%), lowering total operational expenditures (OpEx) by roughly 15–20%.**
- **For the local health system: Producing localized antivenoms reduces import tariffs, long-distance cold-chain transport costs, and emergency inventory shortages.**

8. Risk Reduction

We mitigate biological, operational, and commercial risks across our client base:

- **Quality and consistency guarantee: Every batch of raw venom is paired with a certificate of analysis detailing purity, concentration, and biological activity. If a batch does not meet agreed-upon thresholds, it is replaced free of charge.**
- **Continuity guarantee: We mitigate supply risks through 1–3 year contracts that insulate clients from wild market volume fluctuations and stock-outs.**
- **Farm safety guarantee: Our industrial safety protocols (designed by Mahdi Bahiou) ensure the annual farm sting rate drops below 0.1%. Facilities feature a dedicated emergency intervention room supervised by nurse Wassim Touiza equipped with ready-to-use antivenom.**
- **Device guarantee: Hardware features a comprehensive two-year warranty backed by remote technical support.**

9. Accessibility

We make advanced bioprocess technology accessible to wider, traditionally underserved segments:

- **Standalone offline operation: Scorpion breeders in remote regional areas with poor internet connectivity can deploy the device fully offline to receive immediate on-site recommendations.**

- **Website localized access: The platform is fully localized in Arabic, English, and French, supporting international electronic payment systems for global and local orders.**
- **Local distribution channels: Raw venom and pharmaceutical preparations are routed efficiently through standard public hospital networks and commercial pharmacies, with rapid delivery established for major municipal hubs.**
- **Targeted training infrastructure: We offer short training courses (both online and in-person) to teach farmers how to set up the device and interpret stochastic reports, making high-level technology usable for non-specialists.**

10. Convenience / Usability

We simplify highly complex technical actions into user-friendly steps:

- **Hardware plug-and-play: The device runs on a standard rechargeable battery and features a single physical power button. Once active in the farm space, it automatically prints actionable messages like *"Milking recommended in X days"* or *"Increase temperature to 32°C"*. Users do not need to understand the underlying mathematics.**
- **Website data entry: Uses a clean drag-and-drop interface (e.g., sliding temperature controls, entering basic scorpion counts, selecting feed parameters from dropdown menus). A single *"Generate Report"* button produces a clean, print-ready PDF.**
- **Streamlined product ordering: Ordering raw materials or derived cosmetic products is facilitated through automated web forms or an automated WhatsApp chatbot that guides the client step-by-step.**
- **Technical support hotlines: A dedicated WhatsApp hotline answers inquiries during local working hours, integrated with an online knowledge base containing short video tutorials.**



AXIS 3 : STRATEGIC MARKET ANALYSIS

1. Market Sector Presentation

Potential Market

The potential market for our project consists of every individual or institution that needs raw scorpion venom, derived products (antivenoms, oncology drugs, cosmetic preparations), or consulting and digital services related to scorpion breeding and venom production.

This market comprises the following categories:

Category	Entities Involved	Demand Drivers
International and local pharmaceutical companies	Sanofi, Pfizer, Merck, as well as Algerian antivenom manufacturers	Need for pure venom in stable quantities to produce antivenoms, cancer drugs (Chlorotoxin), and painkillers.
Scientific research centres and laboratories	Universities (e.g., Yahia Fares University of Médéa, University of Science and Technology), Pasteur Institutes, toxinology labs	Need for raw venom or purified peptides for R&D, and studying the genetic composition of venoms.
Cosmetic manufacturing companies (skincare)	Local and regional cosmetic companies (e.g., Laboratoire MA Laboratories, Belle Etoile)	Need for venom peptides as active ingredients in anti-ageing creams, as studies have proven their efficacy in improving skin elasticity.
Scorpion breeders (individuals and small companies)	Owners of small and medium-sized farms in Algeria (approximately 20-30 farms)	Need for optimised consulting reports and electronic devices to improve farm productivity and reduce risks.

Key drivers of potential market demand:

- Global scarcity of pure scorpion venom (the most expensive liquid in the world).

- Absence of organised, documented local production in Algeria, despite the abundance of scorpions.
- Increasing demand for effective antivenoms due to the high incidence of stings in desert regions.
- Global trend toward using venom peptides in cosmetics and smart drugs.

Location of the potential market:

- **Locally:** Algerian provinces where scorpions are prevalent (Msila, Djelfa, Laghouat, Biskra, Ouargla), as well as economic centres (Algiers, Oran, Constantine).
- **Regionally:** North African countries (Tunisia, Morocco, Libya, Mauritania) and Arab countries.
- **Globally:** Europe (France, Germany, Switzerland), North America, and Asia (China, India) where the largest pharmaceutical and cosmetic companies are located.

Estimated size of the potential market:

- Major global pharmaceutical companies: more than 50 companies interested in venoms.
- University and independent research laboratories: at least 200 laboratories in the region.
- Local and regional cosmetic factories: about 30 factories in Algeria and North Africa.
- Registered scorpion farms in Algeria: 20-30 farms.

Target Market (Selected Segment)

Initially, we will focus on two main segments, then gradually expand:

- Small and medium scorpion breeders in Algeria (20-30 farms).
- Local and regional pharmaceutical companies and research laboratories (5-10 entities).

After proving feasibility, we will target global and local cosmetic companies.

Products and services offered to each segment:

Segment	Product / Service
Scorpion breeders	Optimised consulting reports ($500-1,500), standalone electronic device, monthly subscription, training.

Segment	Product / Service
Pharmaceutical companies and laboratories	Pure raw venom with certificate of analysis (at globally competitive prices), long-term supply contracts.
Cosmetic factories	Freeze-dried venom peptides (semi-finished ingredient), finished creams under our brand or as white-label products.

Justifications for Selecting this Target Market

- **Confirmed and unsaturated demand:** The scorpion venom market suffers from a global supply shortage relative to high demand, especially from major pharmaceutical companies competing for the small available quantities.
- **No local competitor using mathematical modelling and AI:** All existing Algerian farms operate with traditional methods (trial and error), making our products and services unique.
- **Extremely high price (up to $10,000 per gram):** Producing a relatively small quantity can generate huge revenues that quickly cover investment costs.
- **Natural availability of scorpions in Algeria:** We do not need to import scorpions or create an alien environment, which lowers initial costs and facilitates expansion.
- **Availability of a multidisciplinary team:** We have competencies in mathematics, pharmacy, nursing, industrial safety, and microbiology, ensuring successful implementation.
- **Government support for startups and innovative enterprises:** We can benefit from the advantages (funding, incubators, tax exemptions) that Algeria grants to innovative projects.

Possibility of Signing Purchase Contracts with Key Customers

Based on our preliminary study, there is a strong possibility of signing purchase contracts with:

Potential Customer	Type of Contract	Estimated Quantity	Timeframe
Pasteur Institute of Algeria	Supply of pure raw venom for local	100 grams / year	6 months after production start

Potential Customer	Type of Contract	Estimated Quantity	Timeframe
	antivenom production		
Sanofi (Algeria branch)	Raw venom for pre-clinical trials	20 grams / year	9 months
Central Laboratory of Toxin Analysis (Ministry of Health)	Reference peptides for equipment calibration	5 grams / year	12 months
Yahia Fares University of Médéa (research laboratory)	Raw venom at researcher discount + consulting reports	10 grams / year + 5 reports	Immediately after platform launch (Month 4)
Large scorpion farms in Msila Province (3 farms)	Electronic devices + annual subscriptions	5 devices + 3 subscriptions (first year)	Immediately after prototype development (Month 3)

Strategy for signing contracts:

- Offer free trial devices or reports to the first 3 farms to build reputation and references.
- Participate in international exhibitions (e.g., CPhI Worldwide, Maghreb Pharma) to meet major buyers.
- Promote through the website and professional channels (LinkedIn), attaching certificates of analysis for the first samples.

2. Competition Analysis (Measuring Competitiveness)

In this section, we identify direct and indirect competitors, estimate their numbers and market shares, and analyse their strengths and weaknesses.

A. Direct Competitors

Direct competitors are those offering the same products or services (raw scorpion venom, consulting reports, electronic devices to assist production) in the Algerian or regional market.

Competitor	Product / Service	Estimated Market Share	Strengths	Weaknesses
Traditional scorpion farms *(5-7 farms in Algeria)*	Uncertified raw venom	70% of local raw venom	• Market presence for years • Local relationships with some pharmacists	• No mathematical modelling (guesswork) • Inconsistent quality • No certificates of analysis • No devices or consulting reports
Foreign importers *(from Tunisia or Europe)*	Pure (certified) raw venom	30% of local market (small volume due to customs)	• High quality • International certificates of analysis	• Very high price (shipping + customs) • Long delivery times • No consulting reports or devices
Foreign remote consulting companies *(very few)*	General consulting reports (not specific to Algerian scorpions)	Less than 5% of the consulting market	• Global brand name	• Do not understand the local environment of Algerian scorpions • High prices in foreign currency • No electronic devices

B. Indirect Competitors

Indirect competitors offer substitutes for our finished products (imported antivenoms, foreign cosmetic creams, general animal breeding consulting services).

Competitor	Substitute	Market Share	Strengths	Weaknesses
Imported antivenoms *(Sanofi, etc.)*	Substitute for local antivenoms	95% of Algerian market	• Globally approved • Strong brands	• Very expensive • Not specific to local strains • Rely on foreign venoms
Global anti-ageing cosmetic creams	Substitute for our creams	90% of the luxury market	• Global reputation • Massive marketing	• Do not contain Algerian scorpion venom peptides • Prohibitively high price • Not readily available in Algerian pharmacies
General agricultural consulting offices	Animal breeding reports	Small share in scorpion breeding	• Experience in agriculture	• Not specialised in scorpions • Do not use stochastic models or AI

C. Number of Competitors and Their Market Shares (Estimate for the Current Year)

Market	Number of Direct Competitors	Their Total Market Share	Our Target Share in Year 1
Raw venom in Algeria	7 farms + 3 importers	100% (we are not currently present)	10% of local market (raw venom)

Market	Number of Direct Competitors	Their Total Market Share	Our Target Share in Year 1
Consulting reports	2 foreign entities (remote)	100%	60% (because the service is new)
Electronic devices	No competitor	0%	80% (we are innovators)

D. Our Competitive Advantages

- **Stochastic mathematical model:** No competitor possesses it. It increases production by 31% and reduces costs.
- **Standalone (offline) device:** The first of its kind on the market, battery-powered, providing instant recommendations.
- **Optimised reports:** Contain dynamic graphs and extinction risk analysis – a service that does not exist elsewhere.
- **Vertical integration:** We produce raw venom, develop drugs and creams, whereas local competitors sell only raw venom.
- **Competitive price of finished products:** Our antivenoms and creams will be 30-40% cheaper than imported ones.
- **Multidisciplinary team:** Combines mathematics, pharmacy, nursing, safety, and microbiology.

3. Marketing Strategies

Marketing Mix (4Ps)

1. Product

- Pure raw venom with a digital certificate of analysis (QR code).
- Standalone electronic devices to calculate optimal production conditions.
- Optimised consulting reports (PDF with graphs).
- Local antivenoms specific to Algerian scorpion strains.
- Anti-ageing creams containing venom peptides.
- Monthly / annual subscriptions for updates and technical support.

2. Price

- **Raw venom:** We maintain a high price ($10,000 per gram) because it is a rare, valuable product – we do not reduce it.
- **Consulting reports:** $500-1,500 depending on farm size.
- **Electronic device:** Cost price (about $30) plus a small margin, with a Freemium model.
- **Monthly subscription for breeders:** $50 (includes unlimited reports and updates).
- **Antivenoms and creams:** 30-40% lower than imported alternatives, to make them accessible to the local market.

3. Place (Distribution)

- **Website:** B2B and B2C platform with international shipping.
- **Standalone device:** Sold via the website and also through local distributors in major provinces.
- **Presence at trade fairs:** CPhI Worldwide, Maghreb Pharma, Algerian innovation fairs.
- **Partnerships with pharmacies and hospitals:** To distribute antivenoms and creams locally.
- **Representative offices in Algiers and Oran** to serve major customers.

4. Promotion

- **Digital marketing:** Targeted ads on LinkedIn, Google Ads, and specialised biotechnology pages.
- **Free content:** Publish case studies and comparisons between traditional methods and our model.
- **Free trial offers:** The first 3 farms receive a free device + first free report in exchange for a testimonial.
- **Direct outreach to major buyers:** Via dedicated email and calls, attaching certificates of analysis for initial samples.
- **Webinars:** To teach scorpion breeders how to use the device and interpret reports.
- **Partnerships with business incubators and universities:** To market academic products (raw venom at a discount for researchers).

Considering Financial Capabilities

Because we are in the start-up phase, we will rely on:

- Partial self-financing from the team's savings.

- Support from business incubators (in exchange for a small equity stake or interest-free).
- National innovation grants (Algerian startup support fund).
- Freemium model making the device low-cost, with revenues from subscriptions and paid reports.

Balanced Marketing Mix

We will not focus on a single tool; rather, we will balance:

- Continuous product improvement based on customer feedback.
- Competitive pricing that achieves a reasonable profit margin while being accepted by the market.
- Wide and fast distribution through digital and local channels.
- Strong promotion that builds trust in our brand as a reference in the scorpion venom field.



AXIS 4 : PRODUCTION AND IMPLEMENTATION PLAN

1. Production Process

The production process for scorpion venom (raw and refined), pharmaceutical and cosmetic derivatives, as well as electronic devices and consulting reports, goes through several sequential stages. We explain them here in a systematic way to ensure quality and traceability.

Stage 1: Procuring Raw Materials and Preparing the Environment

Activity	Description	Quality Criteria
1.1 Procuring scorpions	Purchase healthy scorpions of Androctonus or Hottentotta species from licensed farms or from nature with a permit.	Health check by the microbiologist (Dr. Khames), disease-free, genetic diversity.
1.2 Preparing the farm	Climate-controlled rooms (32.6°C, 50-60% humidity), ventilation, appropriate lighting.	Application of hygiene and safety protocols (supervised by El Mahdi Bahiou).
1.3 Procuring feed	Purchase locusts, crickets, or protein-rich artificial feed.	Feeding at 88% of satiety (Nu^* = 0.88$) according to the mathematical model.
1.4 Procuring packaging materials	Sealed glass vials (1ml, 5ml, 10ml), sterile cream containers, vacuum bags.	Pharmaceutical standards (USP / EP).
1.5 Procuring chemical components and analytical means	Solvents (distilled water, acetonitrile), HPLC columns, reagents.	Analytical grade purity (HPLC grade).

Stage 2: Raw Venom Production (Milking and Collection)

Activity	Description	Quality Control
2.1 Assembling the electronic device	Programming the ESP32 / Raspberry Pi with the stochastic model (birth-death processes, Gaussian closure).	Functional test to ensure correct calculation of T^*, Nu^*, γ, and m.
2.2 Monitoring environmental conditions	Real-time reading of temperature and humidity, issuing recommendations.	Temperature must be $32.6 \pm 1.0°C$
2.3 Milking by electrical stimulation	Use an electrical stimulator (18V, 45Hz, 10% duty factor) on the scorpion for 5-10 seconds.	Milking every 11 days (γ = 0.09$), removing 6 units ($m$ = 6).
2.4 Collecting the venom	Collect drops in a sterile pipette, transfer to a sterile, refrigerated glass vial.	Maintain a cold chain (2-8°C) immediately.

Stage 3: Purification and Stabilisation (In the Laboratory)

Activity	Description	Quality Control
3.1 Centrifugation	Remove cellular debris (centrifuge at 10,000 rpm for 15 minutes).	Discard pellet, keep supernatant.
3.2 Filtration	Filter through a 0.22 µm membrane to sterilise the venom.	Sterility check (culture sample on appropriate medium).
3.3 Lyophilisation	Freeze-dry the venom to obtain a pure powder (with a cryoprotectant).	Residual moisture < 2%.

Activity	Description	Quality Control
3.4 Chemical analysis (HPLC / MS)	Analyse peptide composition, detect presence of chlorotoxin.	Must show 12 peptide peaks (MALDI-TOF).
3.5 Batching	Package the powder into small vials under an inert gas.	Label each batch with a unique number and QR code.

Stage 4: Manufacturing Finished Products (Pharmaceuticals and Cosmetics)

Activity	Description	Quality Control
4.1 Manufacturing antivenoms	Dilute and modify the stabilised venom with formaldehyde to produce toxoid, then inject into animals to produce antibodies. *(Animal steps may be outsourced.)*	Neutralisation assay for efficacy.
4.2 Manufacturing anti-ageing cream	Dissolve peptides in a cream base (water, oil, emulsifier, permitted preservatives).	Stability test (different temperatures, 3 months).
4.3 Manufacturing semi-finished cosmetic ingredients	Freeze-dry peptides and package in sterile bags for sale to other cosmetic factories.	Microbiological analysis (no microbes).

Stage 5: Product Customisation (Tailoring to the Customer)

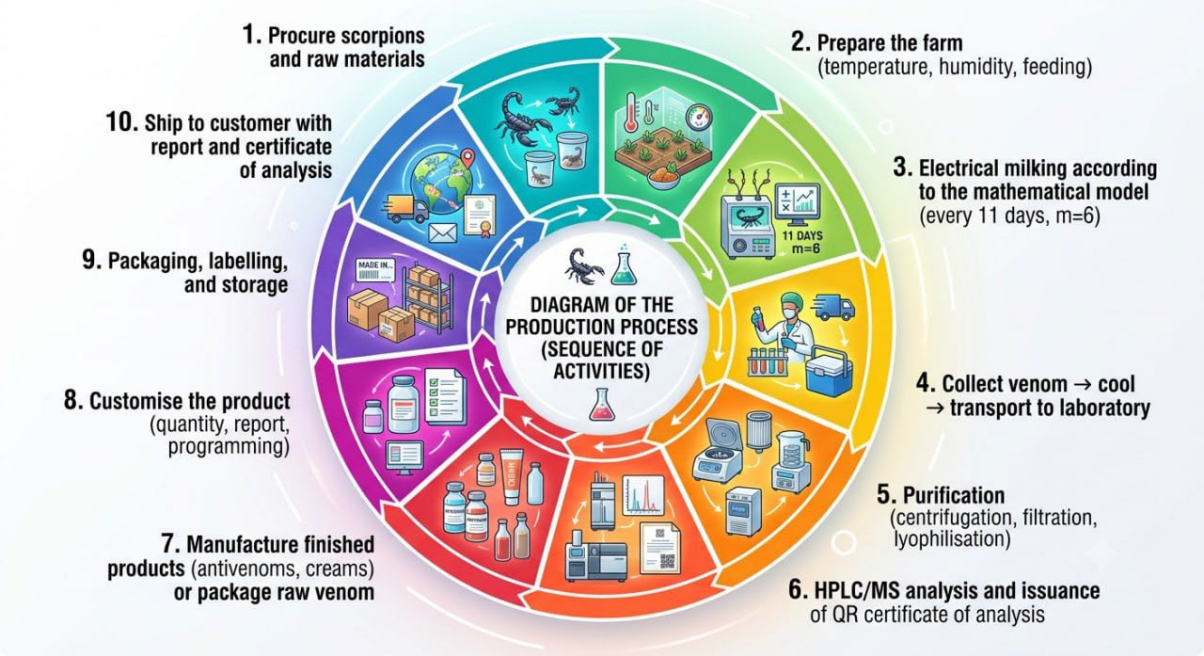
Activity	Description	Acceptance Criteria
5.1 Custom packaging	Package raw venom in small quantities (1 g, 5 g, 10 g) or according to customer request.	Precise weight $\pm$ 0.01g.

Activity	Description	Acceptance Criteria
5.2 Attaching a digital certificate of analysis (QR)	Create a QR code that links to a webpage showing the analysis results for that batch.	Easy verification via mobile phone.
5.3 Preparing consulting reports	Produce customised PDF reports (for farms) containing dynamic simulation graphs.	Must include T^*, Nu^*, γ , m and expected profitability.
5.4 Programming devices on demand	Load a specific mathematical model according to scorpion type and local climate.	OTA update after purchase.

Stage 6: Packaging, Labelling, and Storage

Activity	Description	Quality Specifications
6.1 Packaging raw venom (powder or liquid)	Brown or clear glass vials, airtight, with rubber stopper and aluminium seal.	Packaging at 2-8°C for transport.
6.2 Packaging creams	Aluminium tubes or sealed plastic containers with outer carton.	Shelf life of at least one year.
6.3 Packaging electronic devices	Cardboard box with logo, containing the device, USB cable, quick guide.	Shock protection.
6.4 Labelling	Label with batch data, production date, expiry date, company logo, QR code.	Compliance with local and international legal requirements.
6.5 Storage	Refrigerators (for raw venom), dry shelves (for creams and devices).	Daily monitoring of temperature and humidity.

Summary Diagram of the Production Process (Sequence of Activities)



Quality Assurance at Each Stage

Stage	Responsible Person	Quality Tool
Procuring scorpions	Dr. Khames (Microbiologist / Veterinarian)	Health certificate, vaccination record
Milking and collection	Farm technician + supervisor (Imene Ghernouti)	Adherence to schedule γ, m
Purification and analysis	Imene Ghernouti + Dr. Younes	Record results in batch database
Packaging and labelling	Packaging technician	Weight, seal, and label inspection
Shipping	Logistics officer	Refrigerated shipping (for venom) or standard (for creams/devices)

2. PROCUREMENT

A. Purchase Policy

We adopt a flexible and cost-effective purchasing policy while ensuring the quality of raw materials and equipment. We divide the needs into three categories:

Category	Purchased Items	Purchase Policy
Raw materials	Scorpions, scorpion feed (locusts, crickets), chemicals for analysis (solvents, HPLC columns), packaging materials (vials, seals)	Purchase scorpions from licensed farms or local collectors under seasonal contracts. Purchase feed from insect suppliers. Purchase chemicals from authorised distributors.
Supplies	Cleaning and sterilisation supplies, gloves, syringes, collection tubes, device batteries, electronic boards (ESP32, Raspberry Pi)	Local purchase from medical and laboratory supply vendors. Purchase electronics through reliable platforms (e.g., Mouser, DigiKey) in small quantities initially.
Equipment	Temperature control units, refrigerators, centrifuges, lyophiliser, precision balance, HPLC unit (may be rented initially)	Gradual purchase according to funding. Start with basic equipment (refrigerator, balance, centrifuge), then rent an HPLC or contract with an external laboratory.

B. Key Suppliers

Supplier	Product / Service	Location	Reason for Choice
Local scorpion farms *(3-4 farms in Msila, Djelfa, Laghouat)*	Live scorpions *(Androctonus, Hottentotta)*	Algeria	Geographic proximity, lower cost, support for local economy.
Insect breeders *(locust and cricket breeders)*	Live feed	Algeria *(Blida province, Algiers)*	High feed quality, weekly delivery.

Supplier	Product / Service	Location	Reason for Choice
Biochem *(authorised chemical distributor)*	HPLC solvents, columns, reagents	Algiers *(branch)*	Quality certificates, fast delivery.
Mouser Electronics *(online)*	ESP32, Raspberry Pi, sensors	Global *(ships to Algeria)*	Competitive prices, reliability.
Local laboratory equipment supplier *(e.g., Bio-Rad Algérie)*	Centrifuges, refrigerators, balances	Algiers	Local warranty and maintenance.
External analysis laboratory *(temporary)*	HPLC/MS analysis	Algeria *(e.g., Pasteur Institute)*	Until we acquire our own HPLC.

C. Payment Terms and Delivery Time

Supplier	Payment Terms	Lead Time	Notes
Scorpion farms	30% advance, 70% on delivery	7 days	Seasonal, renewable contracts.
Insect breeders	Monthly payment after delivery	3 days	Weekly delivery.
Chemical distributor	Cash on delivery (COD)	5-7 days	Provides official invoices.

Supplier	Payment Terms	Lead Time	Notes
Mouser Electronics	Credit card or bank transfer in advance	15-20 days	Due to customs. Order 3-month stock to reduce shipping.
Local lab equipment	50% advance, 50% on installation	10-15 days	Includes installation and training.

3. WORKFORCE (HUMAN RESOURCES)

A. Number of Job Positions Created

We plan to create 23 jobs by the end of the second year, distributed as follows:

Department	Year 1	Year 2	Year 3
Administration & Marketing	2	3	4
R&D *(mathematical modelling, AI)*	1	2	3
Production *(breeding, milking, packaging)*	2	4	6
Laboratory & Quality	1	2	3
Safety & Hygiene	1	1	2
Technical support *(devices & reports)*	1	2	3
Logistics & Delivery	0	1	2
Total	**8**	**15**	**23**

B. Nature and Type of Workforce Required (Skills & Qualifications)

Position	Educational Qualification	Required Experience	Work Location
Project Manager	Master / PhD in Mathematics or Business Administration	3 years in innovative projects	Headquarters (Algiers)
Mathematical modeller / Programmer	Master in Mathematics or Computer Science	Algorithms, Gillespie, Python	Headquarters or remote
AI Specialist	Master in AI	DRL (PPO/SAC), TensorFlow	Headquarters or remote
Scorpion breeding officer	Agricultural technician or Zoology diploma	1 year in arthropod breeding	Farm (Msila / Djelfa)
Milking & collection officer	Laboratory technician or veterinary assistant	In-house training	Farm
Purification & analysis specialist (lab)	Bachelor / Master in Biochemistry or Pharmacy	HPLC, MS, sterilisation	Laboratory (headquarters)
Safety & hygiene officer	Industrial safety technician	Biosafety procedures	Farm and laboratory
Web / UI developer	Bachelor in Computer Science	Django, JavaScript, databases	Headquarters or remote

Position	Educational Qualification	Required Experience	Work Location
Marketing / Sales	Bachelor in Marketing or Business	B2B, digital platforms	Headquarters (Algiers)

C. Possibility of Subcontracting

Yes, we will subcontract in the following areas to reduce initial costs and focus on our core competencies:

Area	Reason for Subcontracting	Potential Subcontractor
Advanced HPLC/MS analysis *(temporary)*	High equipment cost ($50,000+)	Pasteur Institute of Algeria, or a central laboratory in Algiers
Antivenom production *(animal steps)*	Requires animals (horses, rabbits) and veterinary licenses	Local pharmaceutical company with an antivenom unit, or a specialised laboratory
Report printing and advanced packaging	We do not have industrial printing equipment	Local specialised printing houses
Delivery and international shipping	We do not want to build a delivery fleet	Shipping companies (DHL, FedEx, EMS Algeria)
Website development *(partial)*	To accelerate launch	Local digital services company (for routine tasks)

4. KEY PARTNERSHIPS

We seek to establish strategic partnerships that support the project and contribute to its success, whether through funding, technical support, or value-chain integration.

A. Suppliers

- **Local scorpion farms:** To ensure a sustainable scorpion supply.
- **Chemical and laboratory equipment suppliers:** e.g., Biochem, Bio-Rad Algérie.
- **Electronics suppliers:** Mouser Electronics, DigiKey.

B. Public Bodies

- **Ministry of Higher Education and Scientific Research (Algeria):** For research and export licences.
- **Ministry of Health:** For licensing antivenoms and pharmaceutical preparations.
- **National Office for Plant Protection and Pharmaceutical Products:** For pharmaceutical product registration.
- **University business incubators:** For mentoring, seed funding, and tax exemptions.
    - *Examples:* Yahia Fares University of Médéa incubator, Houari Boumediene University of Science and Technology incubator (USTHB).

C. Laboratories and Research Centres

- **Pasteur Institute of Algeria:** Partnership for venom analysis and antivenom production.
- **University research laboratories:** University of Médéa, University of Science and Technology—for collaboration on improving the mathematical model and providing trainee researchers.
- **Regional toxinology centres:** To identify market needs for antivenoms.

D. Banks and Financial Institutions

- **Local Development Bank (BDL) or Agricultural and Rural Development Bank (BADR):** For a professional bank account and equipment financing.
- **Algerian Startup Support Fund:** For grants or soft loans.
- **National Agency for Support and Development of Entrepreneurship (ANADE):** To support youth and innovative projects.

E. Incubators and Support Structures

- **Business incubator of Yahia Fares University of Médéa:** Mentoring, workspace, and training.

- **"Silicon Badia" incubator** *(if available in Algeria)*: Technical mentorship and investor connections.
- **Chamber of Commerce and Industry:** Assistance with export procedures and licences.

F. Integration Partners

- **Local pharmaceutical companies** *(e.g., Saidal, or others)*: To manufacture antivenoms under their brand (white label) or to purchase raw venom in bulk.
- **Cosmetic factories** *(e.g., Laboratoire MA Laboratories, Belle Etoile)*: To sell semi-finished cosmetic ingredients (freeze-dried peptides) or market creams under their brands.
- **Technology and software companies:** To develop the website and mobile apps (could be an Algerian startup).

G. Distribution and Subcontracting Channels

- **Shipping companies:** DHL, EMS, Yalidine for local and international delivery.
- **Certified export offices:** To assist with customs procedures for foreign customers.



AXIS 5 : FINANCIAL SYSTEM AND PLAN

1.0 Technical Architecture & Local Component Market Valuation

The technical infrastructure of the Organic venture integrates advanced Embedded Systems (Hardware Frameworks) with scalable Cloud computing infrastructures (SaaS models). Based on empirical pricing structures evaluated in the Algerian hardware distribution sector—accounting for localized logistics constraints and microelectronic supply volatility—the direct capital allocation required to manufacture a single automated biological extraction unit has been quantified in Table 1.1.

Technical Component	Unit Cost (DZD)	Quantity (4 Units)	Aggregated Cost (DZD)	Operational Engineering Context
Raspberry Pi 4 (Central Module)	25,000	4	100,000	Central processing gateway for local edge data encryption and synchronization.
ESP32 NodeMCU Module	2,500	4	10,000	Microcontroller dedicated to precise micro-current impulse generation and signal validation.
Industrial Sensor (DHT22)	1,500	4	6,000	Thermodynamic and humidity metrics tracking to prevent biological sample degradation.
LCD/TFT Screen Interface	2,500	4	10,000	On-board local display interface allowing tech personnel real-time field status telemetry.
Chassis & Power Isolation Units	4,000	4	16,000	IP-rated anti-moisture enclosures, surge protectors, and safety groundings.
Specialized Engineering Calibration	5,000	4	20,000	Laboratory precision soldering, custom PCB routing, and wave frequency setting.
Total Combined Hardware Cost			**162,000 DZD**	**Net Unit Manufacturing Cost: 40,500 DZD**

To correct for the native non-linearity of standard microcontrollers at precision voltage boundaries, each system node implements dedicated external analog signal buffers communicating via local high-speed data buses. Figure 1.1 details the topological mapping of the edge-to-cloud telemetry layout.

Figure 1.1: Topological Hardware Block Diagram & Cloud Integration

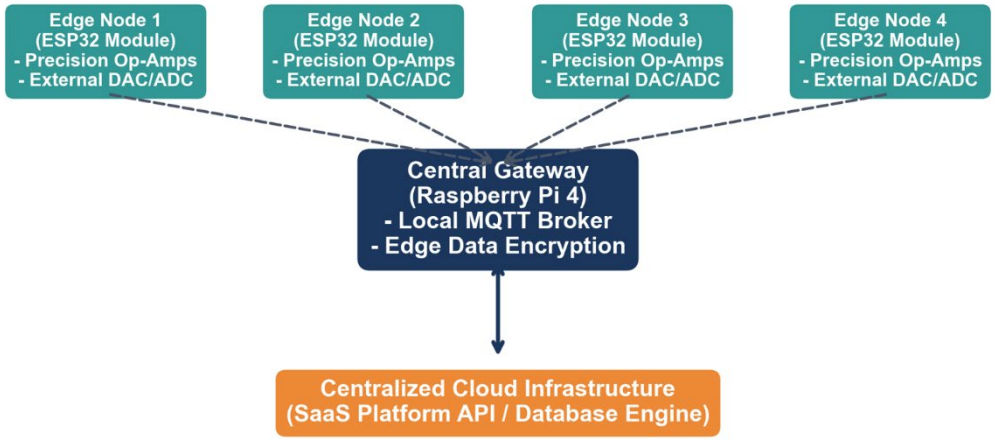


Figure 1.1: Topological block diagram mapping edge nodes to central Raspberry Pi local processing gateways.

> *Technical Directive: System nodes use isolated instrumentation operational amplifiers to preserve signal stability against cross-channel EMI inside the bio-extraction environment.*

2.0 Capital Expenditures & Laboratory Optimization (CAPEX)

Initiating the operations framework demands a baseline capital injection of 1,000,000 DZD. Financial assets are primarily directed toward environmental and structural laboratory optimizations to meet criteria for national sanitary accreditation and official regulatory compliance.

Asset Category	Structural Composition	Financial Value (DZD)	Allocation %
Smart Extraction Hardware	Complete assembly of 4 localized smart extraction devices	162,000	16.2%
Infrastructure & Facilities	Laboratory space lease acquisition and climate-controlled adaptation	350,000	35.0%

Laboratory Equipment	Anti-static workstations, baseline analytical tools, safety infrastructure	60,000	6.0%
Software Platform & Core Cloud	MVP user architecture development, secure API protocols, database setup	235,000	23.5%
Corporate Registry & Legal	Commercial documentation, legal notary representation, and INAPI patent filing	82,000	8.2%
Capital Contingency Reserve	Strategic liquidity pool protecting against short-term market adjustments	111,000	11.1%
Total Capital Outlay	**Total Initial Capital Allocation Vector**	**1,000,000 DZD**	**100.0%**

Figure 2.1: Capital Expenditure (CAPEX) Allocation Matrix

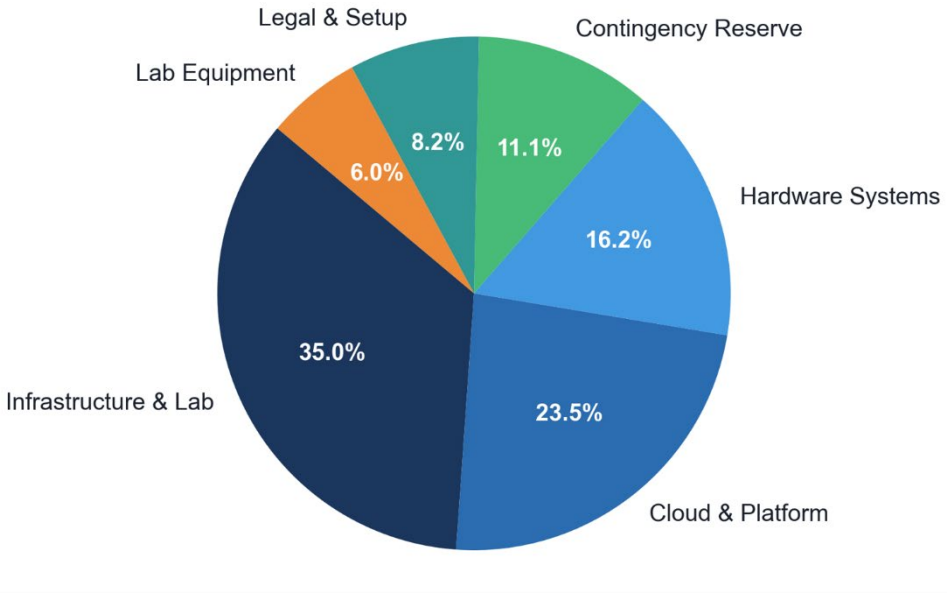


Figure 2.1: Visual Breakdown percentage structures mapping primary target deployment areas for initial financial injection.

3.0 Operating Expenditures & Financial Run-Rate Dynamics (OPEX)

The operational cost matrix is structured elastically to eliminate typical early-stage liquidity constraints. As detailed in Table 3.1, payroll obligations are completely deferred until Month 8, optimizing the operational burn-rate. To explicitly map the execution mechanics behind

the cumulative metric acceleration curves, rolling anticipated operational inflows are modeled directly into downstream evaluation matrices.

OPEX Category	Months 1-3 (DZD)	Months 4-7 (DZD)	Months 8+ (DZD)	Accounting Classification
Facility Lease Obligations	50,000	50,000	50,000	Fixed Operating Expense
Telecommunications & Utilities	20,000	20,000	23,000	Mixed Operating Expense
Biological & Lab Consumables	5,000	15,000	25,000	Variable Operating Expense
Cloud Servers & System Maint.	1,250	1,250	2,000	Fixed Operating Expense
Targeted Account Marketing	0	15,000	30,000	Variable Operating Expense
Hardware Recalibration Cycles	2,000	5,000	10,000	Variable Operating Expense
Specialized Personnel Payroll	0	0	80,000	Variable Operating Expense
Administrative Buffer (5%)	4,000	5,375	11,000	Contingency Fund
Total Monthly Operational OPEX	**82,250**	**111,625**	**231,000**	**System Cost Aggregate**
Modelled Monthly Inflow Vector	**0**	**380,000**	**450,000**	**SaaS Contracts & Leases**

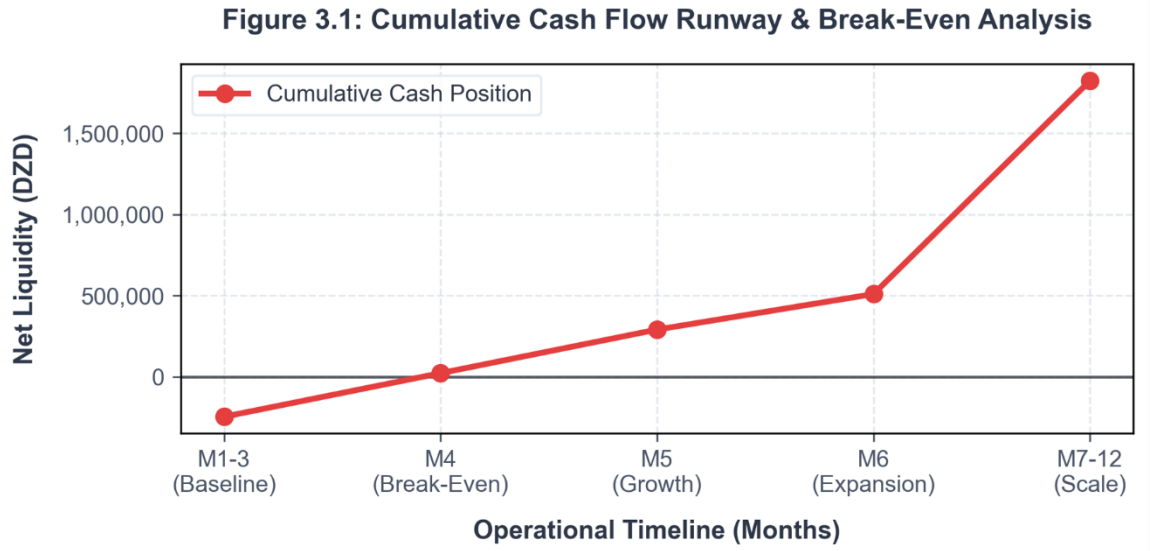


Figure 3.1: Reconciled Rolling Cumulative Cash Runway showcasing transition into positive liquidity fields during Month 4 operations initialization thresholds.

4.0 Revenue Stream Diversification & Macrofinancial Modeling

To counter the prolonged timeline associated with bio-toxin export authorizations, a multi-tiered monetization framework is deployed. Financial stability is secured via short-term SaaS model commitments and localized hardware leasing agreements, bridging the gap prior to full-scale international laboratory bio-export execution in Year 3. Production parameters track volume expansions scaling dynamically out of rolling cash distributions.

Monetization Vector	Year 1 (N+1) [DZD]	Year 2 (N+2) [DZD]	Year 3 (N+3) [DZD]	Volume Scaling Metrics
Cloud SaaS Engine & Data Units	3,050,000	8,000,000	15,000,000	10 to 50 active baseline telemetry server nodes.
Embedded Hardware Sales/Leases	1,500,000	4,000,000	7,000,000	Initial 4 units expanding up to a 50 lease fleet matrix.
Purified Bio-Toxin Material	—	—	8,000,000	Execution phase triggered upon legal export certifications.

Gross Financial Turnover	**4,550,000**	**12,000,000**	**30,000,000**	**Combined CAGR trajectory over evaluation bounds.**

Figure 4.1: Financial Scalability & Yield Projections

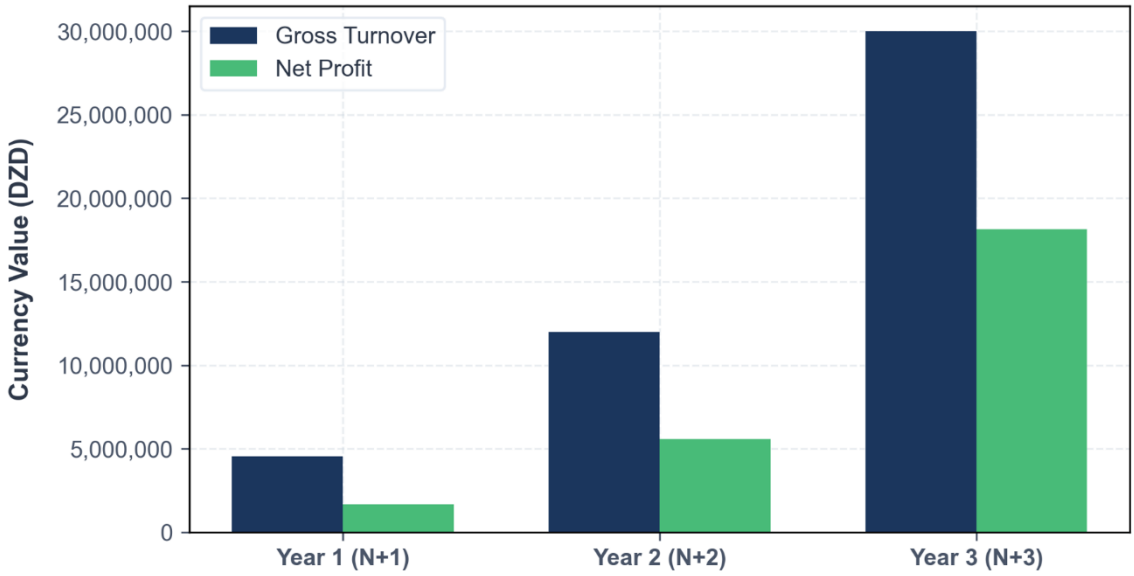


Figure 4.1: Three-year forward revenue expectations demonstrating performance variance scaling alongside volume trajectory updates.

5.0 Pro-Forma Income Statement & Operational Viability Indicators

Table 5.1 tracks corporate performance indices and net margins across the standard three-year horizon. Straight-line asset depreciation metrics are adjusted dynamically to accurately capture ongoing asset growth. This tracks the systematic purchase of additional laboratory instrumentation and fixed machinery funded directly through early retained operating revenues.

Financial Metric	**Year 1 (N+1) [DZD]**	**Year 2 (N+2) [DZD]**	**Year 3 (N+3) [DZD]**
Gross Revenue (Turnover)	4,550,000	12,000,000	30,000,000
(–) Cost of Goods Sold (COGS)	(850,500)	(2,000,000)	(4,500,000)
(=) Gross Profit Margin	**3,699,500**	**10,000,000**	**25,500,000**
(–) Aggregated Operating Expenses (OPEX)	(1,848,250)	(4,020,000)	(6,500,000)
(–) Straight-Line Asset Depreciation (Tracking Growth)	(100,000)	(150,000)	(250,000)

(=) Earnings Before Interest & Taxes (EBIT)	**1,751,250**	**5,830,000**	**18,750,000**
(–) Corporate Taxes (Adjusted for Startup Exemptions)	(91,000)	(240,000)	(600,000)
Net Corporate Profit Retained	**1,660,250**	**5,590,000**	**18,150,000**
Net Margin Efficiency Percentage (%)	**36.5%**	**46.5%**	**60.5%**

6.0 Conclusion and Core Financial Metrics Summary

Break-Even Horizon: The venture successfully crosses the operational break-even threshold within Month 4, registering a resilient, predictable positive cumulative cash flow balance through early dual-track software subscription structures.

Scalability Validation: The rapid expansion of the net corporate margin from 36.5% up to 60.5% across the evaluated 3-year performance cycle validates the low marginal cost structure characteristic of advanced software-driven biotechnology integrations.

First-Year Return on Investment (ROI): The tracking performance indices evaluate Year 1 ROI at 166% relative to the baseline entry requirement vector, demonstrating excellent capital deployment efficiency and operational market alignment.



AXIS 6 : THE FIRST EXPERIMENTAL MODEL

Introduction

SVA (Scorpion Venom Analytics) is an advanced digital platform designed to optimize scorpion venom production using rigorous stochastic mathematical models — specifically birth-death processes — combined with Monte Carlo simulations, sensitivity analysis, real-time environmental optimization, and deep reinforcement learning.
This enterprise startup thesis details a Tri-System Ecosystem engineered to fit varying client operational profiles, budgets, and infrastructural limitations. The system consists of:

1. **System 1: A centralized, pure cloud-hosted SaaS platform.**
2. **System 2: A production-ready, low-power physical IoT prototype powered by an ESP32 microcontroller with an integrated TFT touch HMI and local asynchronous web server.**
3. **System 3: An enterprise-grade, high-compute on-site single-board computer node powered by a Raspberry Pi (currently in active development), serving as a localized, independent mirror of the official platform.**

This comprehensive technical specification provides a structured description of all interfaces, mechanical integrations, data flow matrices, and physical prototyping architectures across all three deployment options.

PART I: SYSTEM 1 — PURE CLOUD SAAS PLATFORM (WEBSITE ONLY)

System Overview

System 1 is the centralized, remote SaaS tier of the Scorpion Venom Analytics platform. It is decoupled from physical hardware modules and operates as a cloud-hosted software application deployed via containerized infrastructure (Docker) on high-availability cloud servers (AWS/DigitalOcean). It acts as the primary heavy computational modeling engine, running parallel simulation paths (10000+ trajectories) over global Wide Area Networks (WAN).

Interface 1 — Hero / Landing Page

General Description
This is the first screen a visitor sees when accessing the SVA platform. It is designed with a clean, professional aesthetic using muted green tones that reflect the biological and scientific nature of the project. Its primary goal is to communicate the platform's value proposition immediately and convert visitors into active users.
Website Official Logo
The SVA platform is represented by an official custom-designed logo that visually embodies the identity of the project. The logo features a geometric low-poly scorpion whose body is shaped into the letters "SVA", combining the brand name with the core biological subject of the platform. Gold-tinted edges run through the teal polygonal structure, reflecting the project's dual focus on scientific precision and economic value. A smaller scorpion is embedded within the letter "A", reinforcing the theme of scale and

biological detail. This logo appears on all official communications, the website header, reports, and platform materials.



Figure 0 — SVA Official Website Logo

Components and Content

- **Navigation Bar (Navbar):**
  - **SVA logo with the full name "Scorpion Venom Analytics" displayed top-left.**
  - **Navigation links: Home - Features - Calculator - Results — enabling fast access to each section.**
- **Hero Title & Subtitle:**
  - **"Optimize Scorpion Venom Production with Precision" — the main headline in bold, large typography.**
  - **Subtitle: "Advanced stochastic modeling platform using birth-death processes and Monte Carlo simulations to maximize your venom yield."**
- **Call-to-Action (CTA) Buttons:**
  - **"Start Free Analysis" — primary green button; routes the user directly to the calculator section.**
  - **"Watch Demo" — secondary outlined button; allows visitors to preview the platform's capabilities before committing.**
  - **Promotional badge: "FREE TESTING OFFER — First 100 analyses completely free" — reduces friction for new users.**
- **Key Performance Metrics:**
  - **35% — Average Yield Increase across optimized farms.**
  - **99.2% — Model Accuracy validated through thousands of simulations.**
  - **10K+ — Simulations per second processed by the platform engine.**
  - **24/7 — Real-time support availability.**

Functional Role

This interface serves as the primary marketing touchpoint. The statistics build immediate scientific credibility, while the dual CTA buttons cater to two visitor types: those ready to start immediately, and those who need more information before engaging.



Figure 1 — Hero / Landing Page Interface

Interface 2 — Core Technology / Features

General Description
Titled "Powered by Advanced Mathematics", this section presents the six core technological pillars underpinning the SVA platform. It targets a scientifically literate audience — researchers, investors, and specialized scorpion breeders — by articulating the mathematical and computational foundations of the system.
Feature Cards

1. **Birth-Death Process:**
    - **Markov chain modeling of venom production and degradation dynamics with realistic biological constraints.**
    - **Captures the stochastic nature of venom accumulation at the individual scorpion level.**
2. **Monte Carlo Simulation:**
    - **Generates 10000+ stochastic trajectories to produce confidence intervals and empirical distributions.**
    - **Enables robust statistical inference about expected yield under varying conditions.**
3. **Real-time Optimization:**
    - **Dynamically optimizes the milking rate (G_opt) while respecting biological welfare constraints.**
    - **Balances production maximization against scorpion health and recovery time.**
4. **Environmental Factors:**
    - **Integrates temperature effects using the Arrhenius law and quantifies the impact of nutrition on production rates.**
    - **Allows simulation of real-world farm conditions including seasonal temperature variation.**
5. **Sensitivity Analysis:**

- Identifies and ranks the parameters with the greatest influence on production using elasticity coefficients.
- Helps breeders focus resources on the most impactful controllable variables.

6. API Integration:
   - RESTful API enabling seamless integration with existing farm management software and IoT devices.
   - Supports automated data ingestion and report generation for large-scale operations.

Functional Role

This interface establishes the platform's scientific authority. By naming specific mathematical methods (Markov chains, Gillespie algorithm, Arrhenius law), it demonstrates that SVA is not a generic analytics tool, but a purpose-built system validated by rigorous scientific methodology.

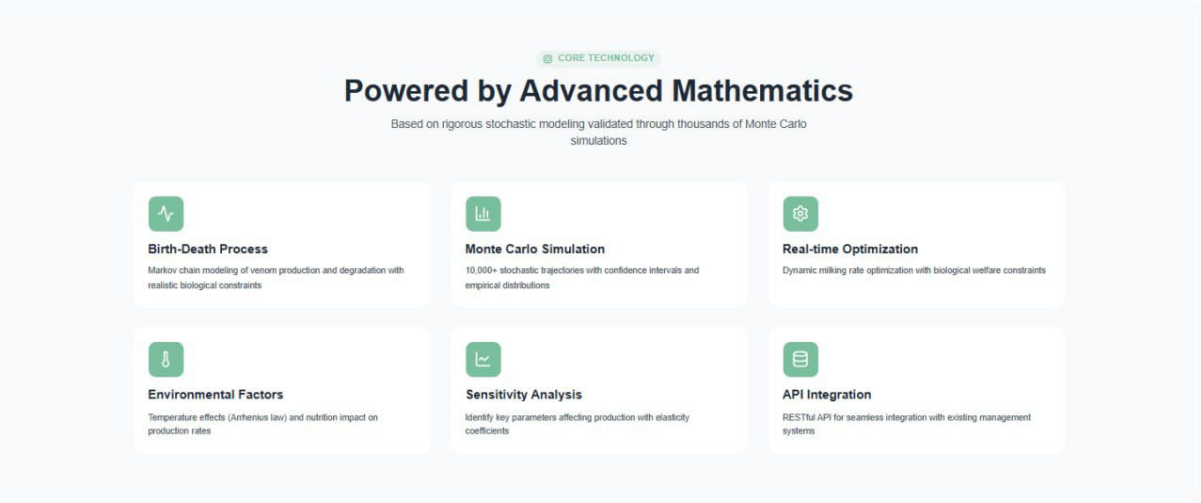


Figure 2 — Core Technology / Features Interface

Interface 3 — Production Calculator

General Description

This is the interactive heart of the SVA platform. Labelled "Production Calculator — Try It Free", it allows users to enter their farm-specific parameters and receive a customized, model-driven analysis report at no cost. The design is intentionally minimal to encourage immediate engagement.

Input Parameters

- **Production Rate L (units/day): Base venom production rate per scorpion per day. Default: 1.8**
- **Degradation Rate M (1/day): Natural venom degradation rate. Default: 0.4**
- **Milking Rate G (1/day): Current milking frequency. Default: 0.1**
- **Temperature (degrees C): Ambient farm temperature. Default: 28 degrees C**
- **Nutrition Level: Feeding intensity relative to ad libitum (1.0 = 100%). Default: 1**
- **Monte Carlo Simulations: Number of stochastic trajectories to run. Default: 1000**

Action Controls

- "Generate Free Analysis Report" — prominent green button; triggers the full mathematical model computation.
- "Reset" — returns all fields to their default values.
- Trust signal: "Your data is encrypted and never shared. No credit card required."

Functional Role

This interface is the conversion point — where visitors become active platform users. The "First analysis free — No commitment" label removes psychological barriers. The six scientifically named input fields signal professional depth while remaining accessible to practitioners.

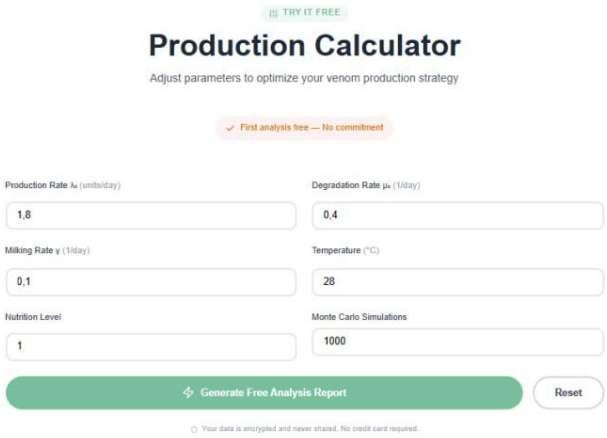


Figure 3 — Production Calculator Interface

Interface 4 — Production Analysis Report (Summary)

General Description

After clicking "Generate Free Analysis Report", the platform computes and displays a structured results page titled "Production Analysis Report — Your Results". It presents the mathematical model's outputs in three progressively detailed panels: key metrics, optimization summary, and sensitivity + Monte Carlo statistics.

Components and Content

- Key Performance Indicator Cards:
  - Expected Yield: 2.04 units per scorpion — the modelled mean venom quantity available for collection.
  - Optimal Milking Interval: 2.0 days — the mathematically derived interval between milking sessions for peak yield.
  - Production Improvement: +114.9% over the baseline unoptimized strategy.
- Optimization Summary Panel:
  - Baseline Yield: 0.1853 units/day — the reference production without optimization.
  - Optimal G_opt: 0.5000 day^-1 — the milking rate that maximizes long-term yield, derived from the birth-death model.

  - **Recovery Constraint: 7 days — the minimum biological recovery window between intensive milking cycles.**
  - **Annual Value Increase: +$574,500 — the projected additional economic value from optimization.**
- **Sensitivity Analysis Panel: Elasticity coefficients showing each parameter's proportional impact on mean venom quantity:**
  - **Production Rate L: +0.6928 (positive — increasing L raises yield proportionally)**
  - **Degradation Rate M: -0.5214 (negative — higher degradation reduces yield)**
  - **Milking Rate G: -0.1714 (negative — over-milking reduces accumulated venom)**
  - **Temperature: -1.1249 (high sensitivity — temperature is the dominant controllable variable)**
  - **Nutrition Level: +0.3079 (positive — better nutrition improves production rates)**
- **Monte Carlo Statistics:**
  - **Simulations Run: 1000**
  - **Mean Final State: 1.88 units**
  - **Standard Deviation: 1.33**
  - **95% Confidence Interval: [1.75, 1.96]**

Functional Role

This panel translates abstract mathematical results into actionable farm management insights. The economic value figure (+$574,500) makes the optimization directly legible to investors and farm owners, while the elasticity coefficients guide practitioners on which parameters to prioritize.

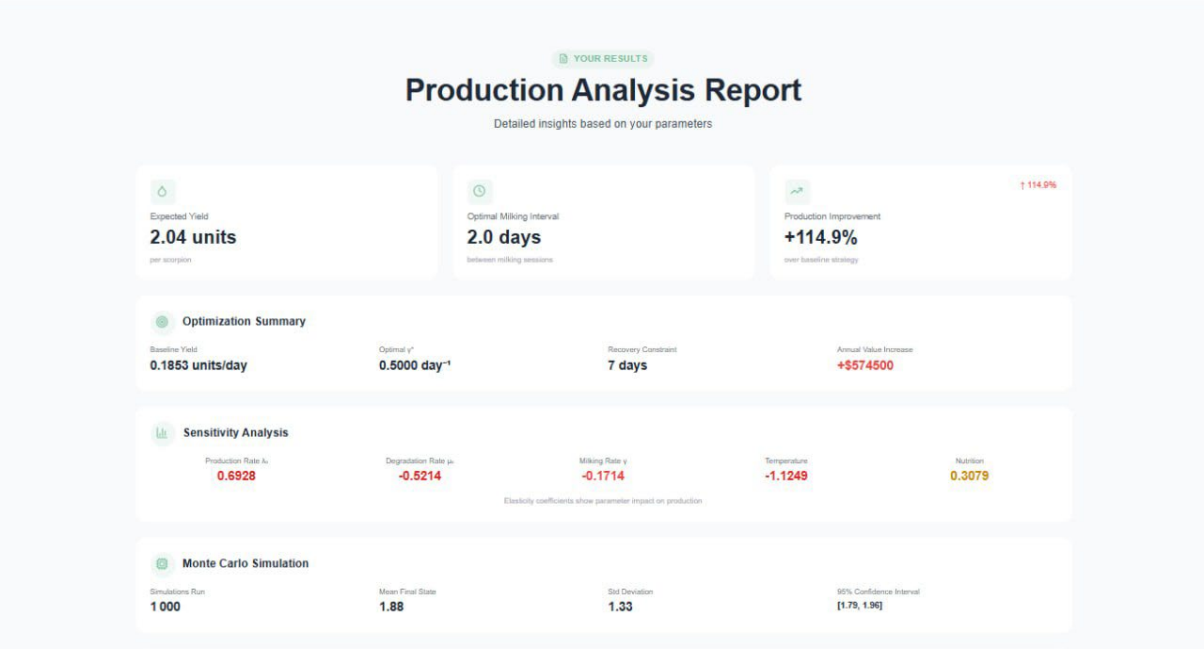


Figure 4 — Production Analysis Report Summary

Interface 5 — Stochastic Process Visualization

General Description

This is the most technically detailed interface of the platform. Under the heading "Scorpion Venom Production: Stochastic Model (Corrected)", it presents six publication-quality charts that give researchers and analysts a complete visual account of the underlying mathematical model.

Components and Content

- **Chart (a) — Single Stochastic Trajectory: Displays one representative birth-death process trajectory over 30 days with biological recovery modelled. Illustrates the natural stochastic fluctuation of venom quantity between milking events.**
- **Chart (b) — Multiple Trajectories with Mean: Overlays multiple stochastic paths with the ensemble mean (red line) and +/- 1 standard deviation band. Demonstrates the spread of outcomes and the convergence of the mean over time.**
- **Chart (c) — Stationary Distribution Comparison: Bar chart comparing the empirical stationary distribution (blue, full model) with the theoretical Markovian distribution (orange). Validates the mathematical model against simulation output; close agreement confirms model correctness.**
- **Chart (d) — Sensitivity to Production Rate L: Shows how the mean venom quantity E[V] changes as the production rate L varies from 0.5 to 3.5 units/day. Linear relationship confirms the model's well-behaved response to this parameter.**
- **Chart (e) — Sensitivity to Degradation Rate M: Shows the decreasing, concave relationship between M and E[V]. Highlights that reducing degradation (e.g., via temperature control) yields disproportionately large gains at low M values.**
- **Chart (f) — Yield Optimization Curve: Plots long-term yield Y(G) against milking rate G, identifying the optimal point G_opt = 0.500 day^-1. The baseline G = 0.1 is shown for comparison, illustrating the 114.9% potential improvement.**
- **Model Information Notes:**
  - **Column 1 assumes immediate recovery (B = 0) — the idealized Markovian case.**
  - **Column 2 includes realistic biological recovery (B = 0.3, K = 0.5) — the corrected model.**
  - **Differences between columns arise from post-milking biological recovery dynamics.**

Functional Role

This interface serves scientific peer review and academic validation. The six charts transform the stochastic differential equations into visually interpretable results, making the model accessible to domain experts, potential academic partners, and regulatory reviewers.

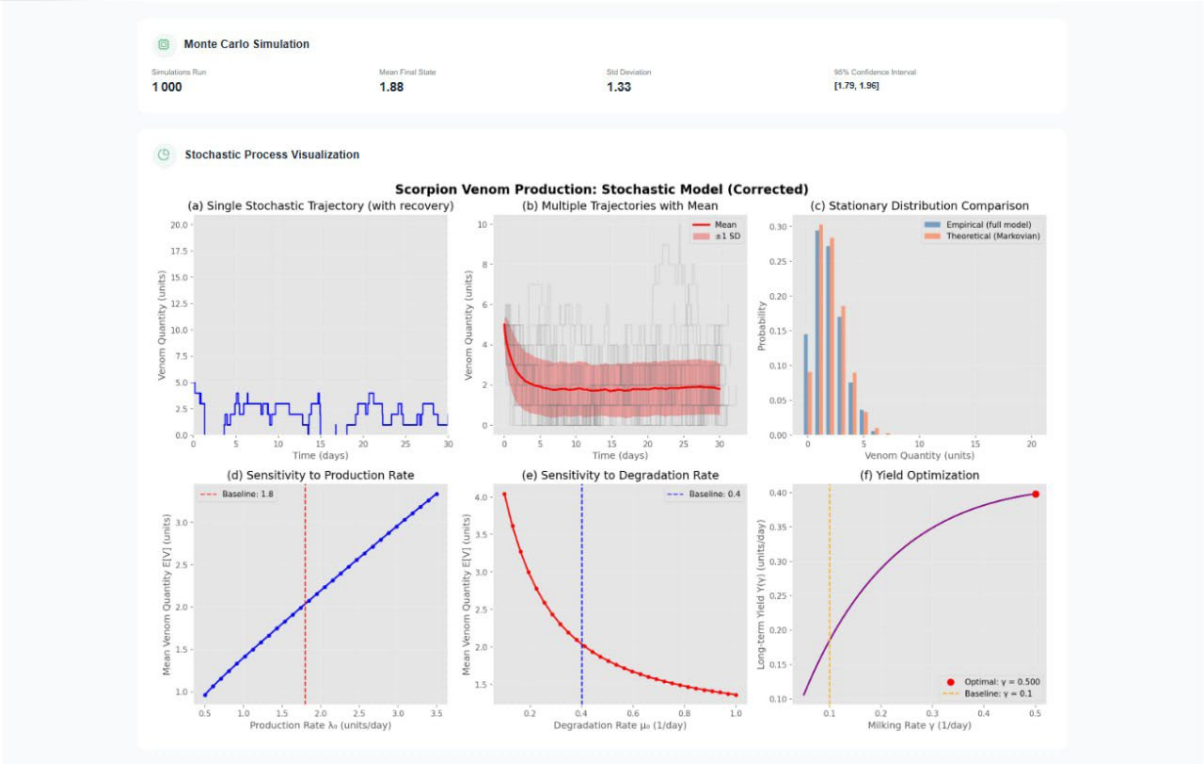


Figure 5 — Stochastic Process Visualization Interface

Interface 6 — Footer & Final Call-to-Action

General Description

The final section of the SVA website combines a commercial conversion prompt with a professional, dark-themed footer. It serves as the closing layer of the user journey: after exploring features, running a free analysis, and reviewing visualizations, users are invited to upgrade to full access.

Components and Content

- **Final CTA Button:**
  - **"Get Full Report & Recommendations" — prominent green button; links to the premium subscription or report download.**
  - **Upgrade notice: "Upgrade to access historical data, batch analysis, and API integration" — clearly communicates what the paid tier unlocks.**
- **Footer — Four Columns:**
  - ***Column 1: SVA Brand* — Logo + full name "SVA — Scorpion Venom Analytics". Tagline: "Advanced stochastic modeling for scorpion venom production optimization."**
  - ***Column 2: Product* — Links: Features - Pricing - API - Documentation**
  - ***Column 3: Company* — Links: About - Blog - Careers - Contact**
  - ***Column 4: Free Trial* — "Start optimizing your venom production today" — reinforces the free entry point. "Try Free" button — maintains conversion opportunity even at the bottom of the page.**
- **Copyright Line: (c) 2026 SVA – Scorpion Venom Analytics. All rights reserved.**

Functional Role

The dark footer provides visual closure and conveys institutional credibility. The final CTA captures users who have scrolled through the entire page — the highest-intent visitors — at exactly the right moment. The four-column footer structure mirrors the layout conventions of established SaaS platforms, reinforcing professionalism.

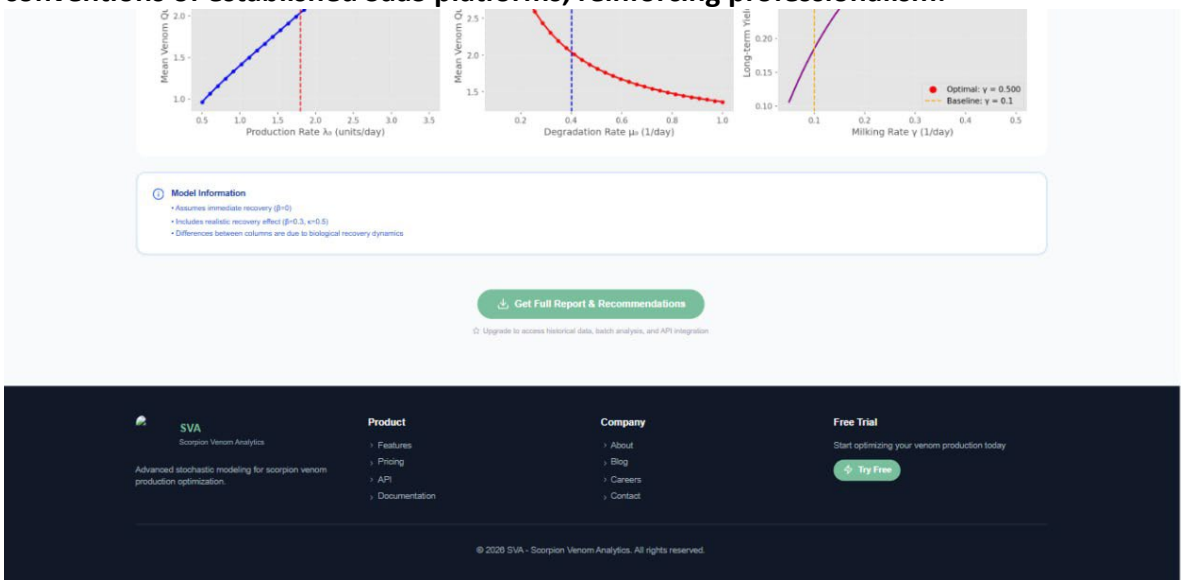


Figure 6 — Footer & Final Call-to-Action Interface

PART II: SYSTEM 2 — ESP32 PORTABLE EDGE NODE (PROTOTYPE FRAMEWORK)

System Overview

System 2 is a pocket-sized, ultra-low-power edge computing module designed to serve as the localized laboratory interface framework. In its current prototype phase, the system operates as a data-driven tracking, simulation, and pulse-recipe calculation engine where all biological and environmental records are inputted manually. However, the underlying architecture is explicitly engineered for future hardware scaling, designed to eventually interface directly with automated electronic stimulation arrays, micro-climate chambers, and digital scales.

Driven by an ESP32 microcontroller housed inside a protective enclosure, it operates entirely offline by establishing a localized Wi-Fi Access Point (SVA_Edge_ESP32). The ESP32 runs an asynchronous C++ web server (ESPAsyncWebServer) serving lightweight, Gzip-compressed (.gz) web interfaces stored within its internal non-volatile LittleFS flash memory partition, allowing operators to connect via a mobile device or laptop to run localized algorithms on-chip.

Physical Enclosure & Assembly Architecture

The physical component consists of a protective 3D-printed case designed to secure the ESP32 microcontroller, its power management circuit (USB-C/battery rail), and an on-board TFT status display screen, leaving open ports for future sensor bus and actuation expansions.



Figure P1_ESP32 — Portable ESP32 Edge Portal Enclosure Assembly

Interface 1 — Local Device Status Dashboard

General Description

Reengineered from the SaaS landing page framework, this interface serves as the primary system management view when an operator connects to the local hardware Wi-Fi network. All consumer marketing copy is replaced with hardware resource diagnostics, core register flags, and peripheral connection readiness maps.

Website Official Logo

The interface layout displays a monochrome, low-overhead version of the official SVA low-poly geometric scorpion logo in the header panel to maintain brand compliance across edge devices.

Figure 0_ESP32 — SVA Edge Official Hardware Logo Element

Components and Content

- **Navigation Bar (Navbar):**
  - **SVA logo with the localized network text string "SVA Local Edge Node [ESP32]" displayed top-left.**

- Navigation links: Status - Calculator - History - Admin — enabling quick access to memory partitions.

- Hero Title & Subtitle:
  - "Local Edge Calculator Telemetry & Memory Status" — main section header.
  - Subtitle: "Standalone pocket node operating under offline edge firmware execution. No active cloud link required."
- Action Diagnostics Control:
  - "Refresh Diagnostics" — button triggers direct microcontroller partition and expansion rail checking.
  - Status banner: "OFFLINE OPERATION — Manual logs buffered locally within internal LittleFS sectors."
- Key Performance Metrics Grid (Real-time Register Reads):
  - ESP32-A1:B2:C3 — Unique Device Hardware MAC Identification address.
  - 4.2V [Battery] — Live analog-to-digital monitoring of the internal battery cell voltage.
  - -45 dBm — Local Wi-Fi hotspot signal strength parameter (RSSI value).
  - SYSTEM READY — Firmware initialization and LittleFS state verification flag.

Functional Role

Provides field technicians with immediate engineering validation that the portable portal's wireless configuration, file system, and battery cell stability are optimal before beginning data processing or testing hardware expansion loops.

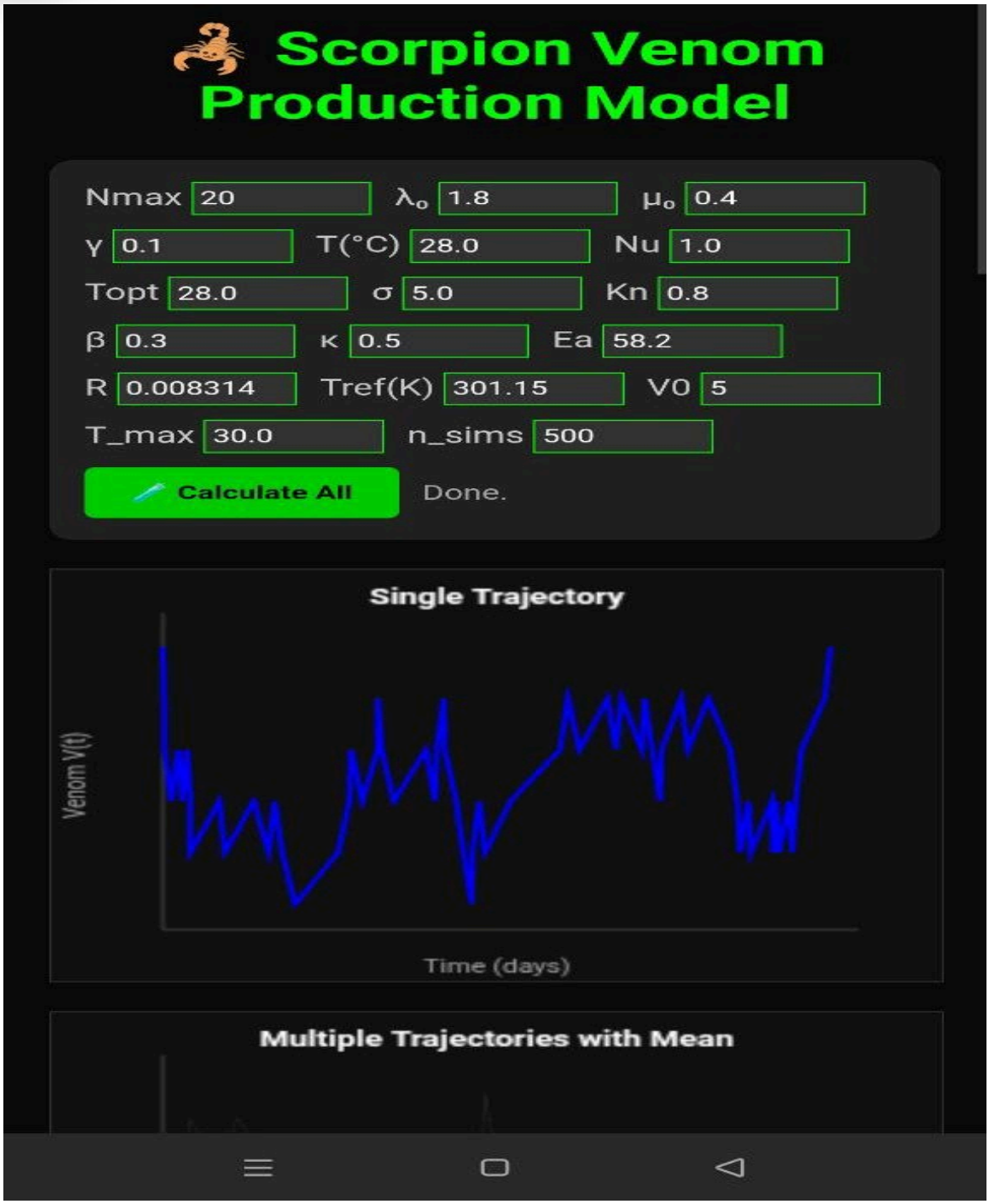


Figure 1_ESP32 — Local Device Status Dashboard Interface

Interface 2 — Embedded Node Constraints Reference

General Description

Titled "Embedded Node Resource Constraints", this static interface notes the physical operational and storage specifications of the ESP32 module. It functions as a minimal data chart to maintain low system memory usage.

Feature Specifications

1. **Memory Limitations: Reports active partition boundaries tracking LittleFS configurations (e.g., Total: 1.5MB, Allocated: 240KB for web assets).**
2. **Asynchronous Architecture: Explains the non-blocking execution loop processing incoming browser requests without stalling mathematical computation or background hardware timer routines.**
3. **Local Static Mapping: Logs default gateway details (192.168.4.1) for explicit network troubleshooting.**
4. **Data Isolation Rules: Documents localized data retention, warning that manual calculation histories risk getting overwritten if local sector thresholds are crossed.**
5. **Fail-safe Profiles: Details software interrupt safety protocols designed to cut power rails automatically if runtime exceptions or future over-current events are detected.**
6. **Asset Compression: Outlines the strict deployment requirement for pre-compressed, raw HTML/JS text strings to fit within the microcontroller's tiny storage budget.**

Functional Role

Acts as a simple, on-board administrative reference map that requires negligible storage space, ensuring memory remains open for active mathematical calculations and future peripheral polling loops.

Interface 3 — Manual Edge Calculator Matrix (Single continuous scroll with inline results)

General Description

The active control dashboard of the portable edge portal, labeled "Manual Edge Calculator Matrix". It replicates the core SaaS calculator form, allowing practitioners to manually key in observed biological parameters and compute the ideal electrical execution recipes designed for future hardware automation layers.

The web page is structured as a single continuous scrolling page. When the user types their variables manually and presses the computation button, the page does not reload; instead, the asynchronous server processes the equations in the background and dynamically appends Interface 4 (Stochastic Synthesis Readout) and Interface 5 (Inline Yield Distribution Plotter) directly beneath the form.

Input Parameters (Manual Fields & Target Actuation Recipes)

- **Observed Production Rate L (units/day): Estimated base venom accumulation rate entered manually. Default: 1.8**
- **Estimated Degradation Rate M (1/day): Assumed systemic natural breakdown coefficient. Default: 0.4**
- **Proposed Milking Interval G (1/day): Target collection frequency parameter to test. Default: 0.1**
- **Target Pulse Frequency (Hz): Software configuration calculating the optimal PWM frequency to be used for future automated extraction. Default: 10Hz**
- **Target Voltage Amplitude (V): Safety amplitude constraint mapping intended output intensity for future hardware loops. Default: 12V**

- Target Temperature Threshold (degrees C): Defines the calculated trigger level for localized climate cooling fans. Default: 26 degrees C

Action Controls

- "CALCULATE ALL & INITIATE PORTAL SUMMARY" — high-visibility green button that executes localized code blocks, computes analytical outputs, validates electrical target profiles, and dynamically appends results matrices and charts further down the page.
- "EMERGENCY DISARM / HALT" — high-contrast red button hardwired to software interrupt sequences to drop simulated system voltages immediately, preserving the safety infrastructure required for future machine deployment.
- System signal: "On-chip mathematical processing loop active. Hardware level safety frameworks armed."

Functional Role

Serves as the primary manual entry and hardware-recipe configuration workspace, enabling immediate verification of production scenarios and electrical target parameters entirely on the ESP32 chip.

Interface 4 — Localized Stochastic Readout (Scrolled Inline View)

General Description

Appears dynamically underneath Interface 3 upon pressing calculate. Titled "Localized Stochastic Readout — Calculation Complete", it visualizes text-based analytics summaries and calculated peripheral execution profiles computed directly by the microcontroller without requiring an internet connection.

Components and Content

- Calculated Statistical Blocks:
  - Expected Model Yield: 2.04 units per scorpion (derived mean statistics computed locally).
  - Recommended Collection Interval: 2.0 days (the estimated extraction timing matrix baseline).
  - Modeled Efficiency Gain: +114.9% increase over unmanaged facility historical data points.
- On-Chip Actuation Parameter Mapping:
  - Baseline Daily Yield: 0.1853 units/day (the unoptimized reference value).
  - Derived Optimal G_opt: 0.5000 day^-1 (the calculated optimal frequency).
  - Target PWM Duty Cycle: 15% calculated pulse-width configuration for future transistor driving.
  - Calculated Session Cutoff: 15s hardcoded maximum execution safety timer profile.

Functional Role

Provides immediate mathematical and mechanical recipe feedback to operators, converting manual variables into clear, descriptive yield numbers and safe hardware operating parameters instantly.

Interface 5 — Inline Yield Distribution Plotter (Scrolled Inline View)

General Description

Positioned at the base of the single-page scroll view under the heading "Inline Yield Distribution Plotter". It renders native graphic curves using highly optimized JavaScript vector code to visually display model outputs and transient pulse waveforms without loading large internet libraries.

Components and Content

- **Lightweight Canvas Chart Layer: A high-contrast vector graph illustrating computed trends:**
  - ***Model Trendline (Blue):*** **Displays the rising curve of venom accumulation over a simulated 30-day timeline based on the manual inputs.**
  - ***Target Waveform Monitor (Red):*** **Illustrates the intended electrical transient wave shape calculated for future automated pulse delivery systems.**
- **Interface Performance Badge: Validates client-side data rendering efficiency: [STABLE] LOCAL GRAPHICS RENDERED [Render Time: 8ms].**

Functional Role

Visualizes mathematical and waveform trends directly in the mobile browser window, giving technicians clear graphical insights from an offline hardware device.



Figure 2_ESP32 — Inline Yield Distribution Plotter Interface

Interface 6 — Hardware Memory Maintenance Utility

General Description
The functional termination section of the local web portal, organizing database maintenance utilities and file clearing controls inside a clear layout.
Components and Content

- **Memory Action Trigger Array:**
  - **"Reboot Edge Node" — executes a soft system reset on the ESP32 chip core.**
  - **"Format LittleFS Memory" — clears local storage sectors, wiping buffered historical calculation logs.**
  - **"Export History Log" — packages saved manual entries as a plain text string for easy copying.**
  - **"Toggle Peripheral Bus I/O" — diagnostic routing switch allowing manual debugging of expansion communication pins.**

- **Hardware Firmware Line:** ***SVA-ESP32-CALC-RevB // Firmware Build 2026-06-v1.0. All Rights Reserved.***

Functional Role

Acts as the main maintenance panel for the edge portal, replacing traditional commercial SaaS links with specialized memory utility and peripheral hardware access controls.

On-Board HMI: Physical TFT Screen Touch Navigation Topology

Simultaneously with the web portal updates, the physical display screen on the protective case mirrors summary data layouts via an SPI-bus interface. The touchscreen uses hardware-interrupt driven navigation to partition views across a two-button capacitive tap environment.

Screen State 0 — Master Processing Home Screen

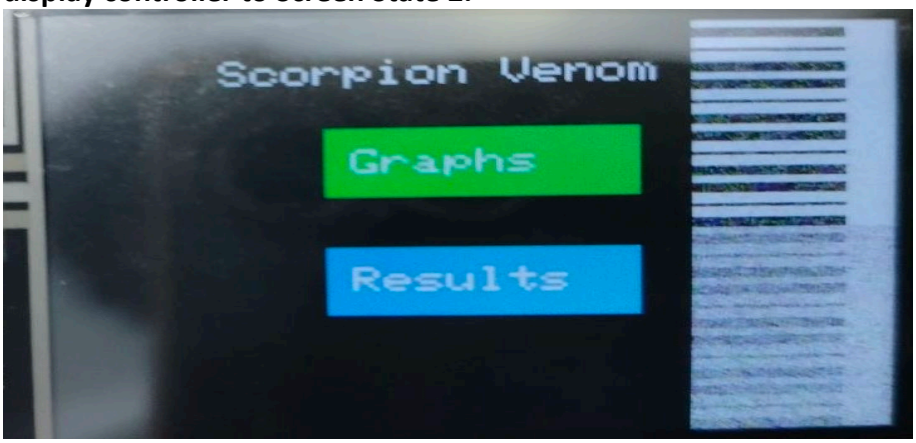
- **General Description: The default menu displayed on the physical case screen once calculation arrays conclude. It serves as the local navigation hub.**
- **UI Components:**
  - **Header String: SVA-Edge-Calc v2.4 // Ready.**
  - **Touch Button 1: High-contrast rectangle icon labeled "RESULTS". Routes the display controller to Screen State 1.**
  - **Touch Button 2: High-contrast rectangle icon labeled "PLOTS". Routes the display controller to Screen State 2.**
  - 

Figure 3_ESP32 — Prototype Screen State 0 (Master Home Screen)

Screen State 1 — Local Analytics Summary View (Results Screen)

- **General Description: Displays text-based analytics summaries directly on the device panel without requiring an external browser connection.**
- **UI Components:**
  - **Data Matrix Array: Clean text lines showing processed yields and environmental numbers:**
    - **Expected Yield: 2.04 Units**
    - **Optimal Interval: 2.0 Days**
    - **Entered Temp: 28.0 degrees C**
    - **Target PWM Res: 10-Bit**

- **Navigation Controller Button: A single full-width touch zone at the base labeled "BACK". Returns the state engine to Screen State 0.**

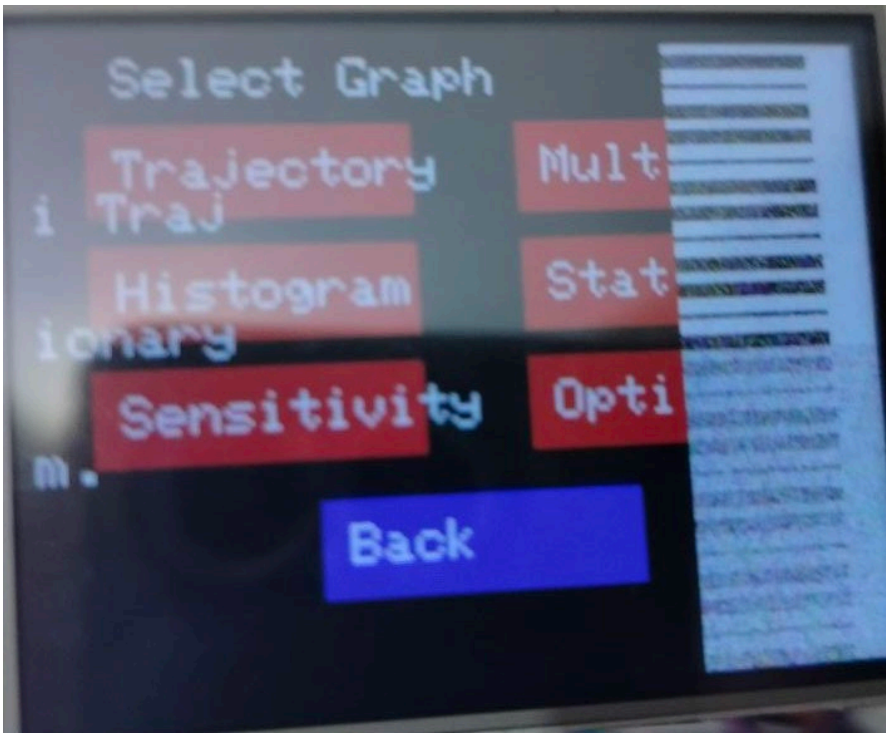


Figure 4_ESP32 — Prototype Screen State 1 (Local Analytics Summary View)

Screen State 2 — Six-Graph Hardware Dash Array (Plots Screen)

- **General Description: Maps the full layout of the 6 core SVA charts into a compressed graphical matrix directly on the physical hardware display.**
- **UI Components:**
  - **Graphic Grid Array: Formats the screen canvas into HTML5 canvas fields rendering native, low-overhead versions of the core charts:**
    - ***Micro-Plot (a):* Single Sample Trajectory Sparkline.**
    - ***Micro-Plot (b):* Ensemble Averaging Waveform Vector.**
    - ***Micro-Plot (c):* Local Stationary Density Bar Histogram.**
    - ***Micro-Plots (d) & (e):* Production (L) and Degradation (M) Slope Derivatives.**
    - ***Micro-Plot (f):* Local Optimization Yield Parabola.**
  - **Navigation Controller Button: A dedicated tap zone at the bottom edge labeled "BACK". Returns the screen state engine to Screen State 0.**

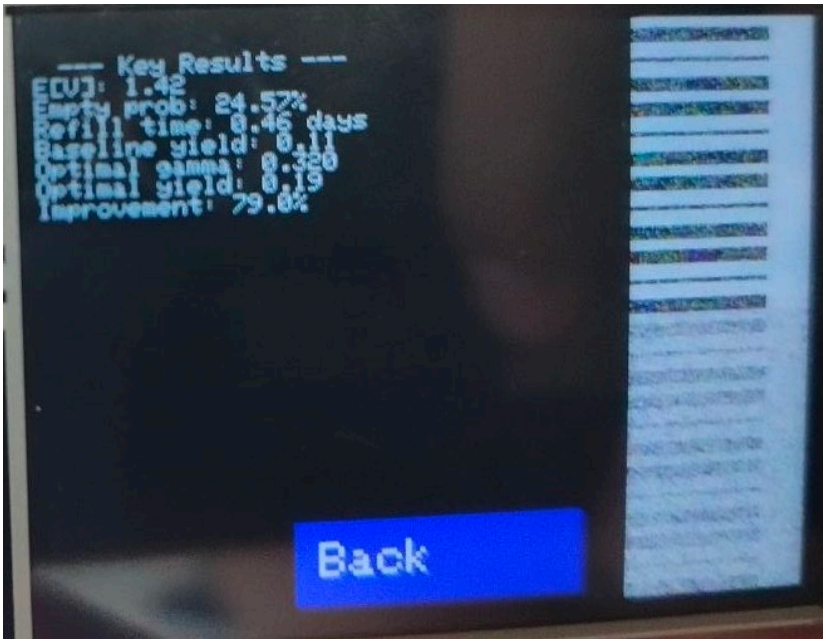


Figure5_ESP32 — Prototype Screen State 2 (Six-Graph Hardware Dash Array)

PART III: SYSTEM 3 — RASPBERRY PI ADVANCED LAB NODE (UNDER ACTIVE DEVELOPMENT)

System Overview

System 3 constitutes the industrial, high-compute on-site tier of the calculator infrastructure. It is designed to run entirely offline, deployed on a Raspberry Pi single-board computer running an embedded Linux platform (Raspberry Pi OS). The local web stack (Nginx reverse proxy, FastAPI/Python backend, and an on-board SQLite/InfluxDB instance) creates an isolated Wide Local Area Network via hostapd and dnsmasq.

Development Status Notice & Automation Roadmap

System 3 is currently under active development. In the current laboratory testing stage, the firmware serves as a high-compute manual simulation sandbox and historical database ledger. However, the system is explicitly architected as a centralized command node. The final active development sprint focuses on embedding multi-device data-linking protocols, enabling this heavy edge node to act as an on-site master server that will automatically ingest telemetry from networked System 2 modules and command downstream laboratory equipment over localized networks.

Physical Enclosure & Development Assembly

The physical unit consists of a high-performance, uncooled enterprise single-board computer development rig, supporting an external solid-state drive (SSD) cluster for local data logging, isolated high-range wireless transceivers, and an integrated power regulation board.



Figure P2_Pi — Physical Raspberry Pi Heavy Hardware Development Assembly

Interface 1 — Enterprise Cluster Management Dashboard

General Description
Visually styled to replicate the System 1 Cloud SaaS Landing Page layout to maintain user experience continuity across enterprise deployments. All consumer marketing prompts are removed and replaced with local network architecture statistics, database states, and hardware resource tracks drawn from the Linux kernel.
Website Official Logo
The interface showcases the geometric low-poly scorpion logo at full resolution, rendered directly from local storage nodes to verify system parity.
Components and Content

- **Enterprise Navigation Bar:**
  - **SVA logo with the specific node descriptor string "SVA Enterprise Server [Station Alpha]" displayed top-left.**
  - **Navigation links: Cluster - Methods - Calculator - Reports - Analytics - System — providing access to core system tools.**
- **Hero Cluster Header & Subtitle:**
  - **"Local On-Site Simulation Cluster & Processing Node" — bold, high-hierarchy title element.**

    - Subtitle: "Enterprise edge computation hub processing localized stochastic simulations and multi-user manual database logging."
- Enterprise Action Cluster:
    - "Launch Cluster Simulation" — primary button directing users to localized calculation sandboxes.
    - "System Check" — secondary utility interface validation key inspecting both software stacks and hardware bus readiness.
    - System notice: "LOCAL DATABASE MASTER ACTIVE — Data securely backed up on encrypted local arrays."
- Local Infrastructure Metrics Grid:
    - 1250 Histories — Total unique calculation profiles logged inside the offline database.
    - 18% / 48 degrees C — Live Raspberry Pi CPU hardware processor core load metrics and thermal status.
    - 14.2 GB Free — Available local storage metrics tracking micro-SD or external solid-state memory banks.
    - 12 Users/Nodes — Count of active local network client leases (such as operators or sub-device portals) connected to this central local routing hub.

Functional Role

Acts as an on-site control tower dashboard. It provides laboratory facility managers with a unified overview of all software sub-systems, active network leases, database health metrics, and future I/O connection slots running within their physical walls.

Interface 2 — Local Technical Documentation & Methods

General Description

Taking advantage of the Raspberry Pi's massive storage capabilities, this interface hosts unpruned technical reference sheets outlining the project's mathematical models entirely offline for on-site research teams.

Documentation Panels

1. Stochastic Equations: Displays complete mathematical formulations of Markovian state changes and biological recovery adjustments.
2. Simulation Parameters: Outlines localized Gillespie path calculation routines running across internal Python libraries.
3. Arrhenius Approximations: Lists thermodynamic coefficient constants adjusted for different scorpion species configurations.
4. Welfare Restrictions: Details the recovery boundary constraints used to protect specimen safety during modeled extraction loops.
5. Elasticity Matrices: Explains the calculation of sensitivity variables across local data blocks.
6. Hardware Integration Endpoints: An integrated, offline interactive API reference portal (e.g., Swagger UI) running directly off the FastAPI server to facilitate custom local data extraction scripts and hardware communication testing hooks.

Functional Role

Provides decentralized research facilities with full technical documentation, algorithmic references, and automation integration endpoints entirely offline, bypassing the need for external internet validation.

Interface 3 — Edge Production Calculator

General Description
An interactive analytical workspace labeled "On-Site Production Calculator & Simulation Sandbox". While maintaining visual styling parity with System 1, its backend is pointed entirely inward to run complex simulation loops across the Pi's processing cores without cloud dependencies.
Form Configuration Parameters (Manual Entry & Targeted Automation Bounds)

- **Production Constant L: Local base generation rate parameter input. Default: 1.8**
- **Degradation Constant M: Local systemic breakdown value definition field. Default: 0.4**
- **Target Extraction Index G: Proposed collection frequency tracker input. Default: 0.1**
- **Target Temperature Input (degrees C): Simulated climate parameter adjustment slider. Default: 28 degrees C**
- **Nutrition Matrix Scaling: Relative feed tracking variable definition box. Default: 1.0**
- **Stochastic Core Iterations: Adjusts local thread processing density (Range: 1000 to 10000 runs). Default: 5000**

Action Controls

- **"Execute Local Stochastic Simulation Pass" — prominent green activation button triggering internal multi-core processing loops.**
- **"Clear Parameters" — resets parameters back to factory system defaults.**
- **Database option check: [X] Commit output data arrays to local historical database ledger for hardware calibration.**

Functional Role
Enables on-site laboratory operators to execute detailed predictive algorithms and test hypothetical ecosystem modifications or automation profiles without transmitting private internal data assets to external networks.

Interface 4 — Localized Analytical Synthesis Report

General Description
The analytical readout screen titled "Localized Production Synthesis — Offline Summary Report". It blends historical manual data logs with local predictive models to provide immediate operational insights and hardware configuration guidelines.
Components and Content

- **KPI Summary Blocks:**
  - **Expected Yield Output: 2.04 units per animal (derived mean statistics computed locally).**
  - **Calculated Optimal Window: 2.0 days (the estimated extraction timing matrix baseline).**
  - **Efficiency Improvement: +114.9% increase over unmanaged facility historical data points.**

- **On-Site Calibration Panel: Displays historical efficiency baselines matched side-by-side with calculated optimal milking frequencies (G_opt = 0.5000 day^-1), pointing out specific local alerts, hardware sync flags, and projected monthly yield valuations.**
- **Sensitivity & Variance Panel: Details elasticity coefficients evaluated from internal database blocks (e.g., Temperature: -1.1249), accompanied by local Monte Carlo confidence parameters indicating standard deviation spreads and target mean states.**

Functional Role

Transforms localized manual data records into high-value operational strategies, allowing operators to optimize environmental parameters and configure target extraction equipment based on internal laboratory data.

Interface 5 — Comprehensive Offline Stochastic Visualizer

General Description

The primary visual analysis dashboard, labeled "Stochastic Model & Historical Trend Analyzer [LOCAL SERVER]". It utilizes native Python graphics or high-performance client-side rendering engines to generate a full six-chart suite using historical data pulled from the Pi's internal database.

The Complete Six-Chart Visual Matrix

- **Chart (a) — Single Stochastic Trajectory: A step-wise timeline tracking an individual sample path over a 30-day window with recovery variables enabled.**
- **Chart (b) — Multiple Trajectories with Mean: An overlaid cloud of parallel sample runs anchored by a bold red ensemble mean line and a shaded +/- 1 standard deviation variance band.**
- **Chart (c) — Stationary Distribution Comparison: A comparative histogram aligning empirical database log models (blue bars) against theoretical probability equations (orange line) to evaluate model drift.**
- **Charts (d) & (e) — Parameter Sensitivity Profiles: Plots identifying production value scaling factors vs. the concave decay trends caused by temperature deviations.**
- **Chart (f) — Yield Optimization Curve: A definitive curve mapping long-term output tracking numbers against extraction parameters (G), indicating maximum efficiency thresholds exactly at G_opt = 0.500 day^-1.**

Functional Role

Provides full academic-grade data visualization capabilities within an isolated laboratory infrastructure, ensuring data privacy while maintaining complete analytical depth.

Interface 6 — Server Configuration & Administrative Utility

General Description

Reengineered from the SaaS pricing footer, this comprehensive, dark-themed dashboard manages advanced configurations for the local Linux operating system, database backups, network routing, and external peripheral hardware bus mappings.

Components and Content

- **Four-Column Enterprise Admin Console:**

    - ***Column 1: Server OS Status*** **— Displays active CPU core load grids, thermal tracking parameters, and system process trees.**
    - ***Column 2: Network Routing Hub*** **— Provides options to modify local Wi-Fi SSID names, manage DHCP range allocations, and configure gateway routing loops for subordinate edge hardware devices.**
    - ***Column 3: Database Utilities*** **— Tools to export secure data records to external storage blocks, run structure evaluations, or clear log tables.**
    - ***Column 4: Automation Hardware Bus*** **— Mappings and diagnostic switches for physical interfaces (SPI, I2C, UART) configured for future laboratory instrumentation hookups.**
- **System Controls Banner: Prominent administrative system commands ("Shutdown Edge Server", "Restart Web Framework Server") backed by explicit copyright rows.**
- **System License Line: *SVA Enterprise Heavy Node OS v3.0 // Node ID: SVA-SER-ALPHA-2026. All Rights Reserved.***

Functional Role

Provides system administrators with complete control over the localized deployment environment, ensuring network stability, secure data handling, and simple configuration interfaces for both current software loops and future hardware expansions.

BUSINESS MODEL CANVAS

<table>
<tr>
<th>Key Partners</th>
<th>Key Activities</th>
<th>Value Propositions</th>
<th>Customer Relationships</th>
<th>Customer Segments</th>
</tr>
<tr>
<td rowspan="3">

Suppliers: Local scorpion farms (M'sila, Djelfa, Laghouat); Insect breeders (Blida, Algiers); Biochem (HPLC solvents); Mouser Electronics (ESP32/Raspberry Pi); Bio-Rad Algérie (lab equipment).
Public & Regulatory: Ministry of Higher Education (licensing); Ministry of Health (antivenom licensing); INAPI (patents).
R&D: Pasteur Institute Algeria (HPLC/MS validation); Univ. of Medea & USTHB (model refinement & interns).
Finance: BDL/BADR (accounts/financing); ANADE (grants/loans).
Incubators: Univ. of Medea Incubator (mentoring/seed funding); Silicon Badia (mentorship/investors).

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Tech R&D: Program stochastic model (ESP32/RPi); Build SaaS website with drag-and-drop reports. (Months 1-3)
Core Operations:

Generate & sell consulting reports (production curves, payback, risk). (Monthly - Year 1)
Assemble/program/ship ESP32 monitoring devices. (On-demand)
Manage Pilot Farm (100-200 scorpions) & manual milking (optimal every 11 days). (Daily)
Lab purification (centrifugation, lyophilization, HPLC/MALDI-TOF) & Certificates of Analysis. (Weekly)

Support & Sales:

Safety/First-Aid training for staff/breeders. (Quarterly)
B2B/B2C Marketing (LinkedIn, Google Ads, CPhI Worldwide).

Key Resources

Pilot farm (100-200 scorpions, temp/humidity controlled)

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Novelty: World’s 1st stochastic (birth-death) model embedded in a device for scorpion milking.
Performance: 31% increase in production (R^2=0.94) via optimal timing (every 11 days, 32.6°C).
Customization: Tailored reports ($500-$1,500); Purity grades (crude to highly purified); Strain-specific peptide concentrations.
Job-To-Be-Done: One-click recommendations for breeders; Stable supply chain of "ready-to-use" raw venom for pharma.
Design: LCD with simple warnings (extinction risk); QR-coded hermetically sealed containers; Drag-and-drop web interface.
Price: Freemium (Device ~$30); Subs ($50/mo); Reports ($500-$1,500);

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Direct Support: WhatsApp groups & hotline for device troubleshooting; Self-service web portal.
Education: Monthly webinars (interpreting model outputs); Email newsletters (updates/offers).
B2B Account Management: Dedicated PM handles pharma contracts; Scientific co-validation (sharing HPLC data).
Contracts: 1-3 year tiered pricing; QR-code batch traceability to digital CoA.
Co-Creation: White-label co-development with cosmetic chemists.
Loyalty: Volume-based rebates for cosmetic companies.

Channels

Website (SEO, Google Ads, SaaS platform)

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Priority 1 - Breeders: 20-30 traditional farms in Algeria (M'sila, Djelfa) lacking technical expertise.
Priority 2 - Pharma: Global/Local labs (Pasteur, Sanofi, Pfizer) needing raw venom for antivenoms, cancer drugs (Chlorotoxin), & painkillers.
Priority 2 - Research: Universities & Pasteur Institutes needing venom for

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➢ *Co-Manufacturing: Saidal (white-label pharma); Laboratoire MA/Belle Etoile (cosmetic ingredients).* ➢ *Logistics: DHL, EMS, Yalidine (shipping); Certified export offices (customs).*	➢ *Lab equipment (centrifuge, freeze-dryer, HPLC, fridge)* ➢ *ESP32, RPi, LCD screens, DHT22 sensors, batteries* ➢ *Core team (8 employees) + external HPLC experts & accountant* ➢ *Proprietary stochastic model (birth-death, Gillespie, R^2=0.94)* ➢ *Source code (C++, Python, JS) & DRL algorithms* ➢ *SVA brand, logo & pending patents*	*Venom sold 30-40% below imports.* ➢ *Cost Reduction: 15-20% less energy/feed for breeders; 25% less purification cost for pharma (high purity).* ➢ *Risk Reduction: Certificate of Analysis (CoA) per batch; Sting rate <0.1%; Extinction alarm.* ➢ *Accessibility: Offline-capable device; Website in Arabic/English/French; Online training.* ➢ *Usability: Plug-and-play, rechargeable; 1-click PDF reports; 24/7 WhatsApp support.*	➢ *LinkedIn B2B content marketing* ➢ *CPhI Worldwide & Maghreb Pharma trade fairs* ➢ *Direct outreach (email/calls with samples)* ➢ *WhatsApp/hotline (24/7 support)* ➢ *Webinars & email newsletters* ➢ *Instant digital PDF delivery* ➢ *DHL/EMS/Yalidine (refrigerated for venom)* ➢ *Local distributors (Algiers, Oran)* ➢ *B2B portal (order tracking, CoA downloads)* ➢	*toxinology research.* ➢ *Priority 3 - Cosmetics: Local/Regional factories (Laboratoire MA, Belle Etoile) seeking anti-aging peptides.*

Cost Structure	**Revenue Streams**
➢ *CAPEX: 1,000,000 DZD (Infra 35%, Software 23.5%, Hardware 16.2%, Legal 8.2%, Equipment 6%, Contingency 11.1%)* ➢ *OPEX: M1-3 (82k/mo), M4-7 (112k/mo), M8+ (231k/mo)* ➢ *Key drivers: Consumables, Payroll (largest), Marketing, Lease*	➢ *Break-even: Month 4 (Y1)* ➢ *Net margin: 36.5% (Y1) → 60.5% (Y3)* ➢ *Y1 ROI: 166% | 3-yr cumulative: 46.55M DZD* ➢ *Y1: 4.55M (SaaS 3.05M + Hardware 1.5M)* ➢ *Y2: 12M (SaaS 8M + Leasing 4M)* ➢ *Y3: 30M (SaaS 15M + Leasing 7M + Venom exports 8M)* ➢ *Breeders: Reports ($500-1.5k) + Subs ($50/mo) + Devices ($30)* ➢ *Pharma: Bulk venom ($10k/gram) + long-term contracts* ➢ *Cosmetics: Peptides + white-label creams (30-40% below imports)* ➢ *Government: Subsidized local antivenoms*

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