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Ministry of Higher Education and Scientific Research
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Department of Mathematics and Computer Sciences



THESIS
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Specialty: Mathematical Analysis and Applications
For the preparation of:
Master degree

Entitled

TEACHING MATHEMATICS IN A MULTILINGUAL CONTEXT IN THE UNIVERSITY LEVEL

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2019-2020

Acknowledgments

In the name of God

Praise to Allah, by whom good deeds are done, which has led us to the path of knowledge and the gates of knowledge, without him none of this might have happened.

I want to thank my supervisor Ms. N. Azrou to her encouragement, her help and her follow-up to finish this work. I also thank the jury members for accepting to discuss and evaluate this dissertation.

I sincerely thank all the students who accepted to collaborate in this research and to my university professors of Dr. Yahia Fares of Medea who accompanied me during my education curriculum.

Finally, a special thank to all my family.

Dedication

I dedicate this modest work to:

My dear father, who supported me so much during my life with his sacrifices.

My dearest mother, who gave me love, patience and support.

My dear sisters, my dear nieces, my dear brother, and all my family and my friends.

Without forgetting all my teachers of primary, middle, secondary and higher education**.**

Table of contents

1/

INTRODUCTION 6
Teaching and learning mathematics in a second or a third language in the world 7
Denmark *8*
France *8*
Cameroon *9*
LITERATURE REVIEW 9
Difficulties of teaching and learning mathematics in second or third language 9
About logical features of language *10*
Quantifiers 10
Mathematical negation 10
Conditional structure 11
Mathematical implication 12
About particular mathematical terms and language *13*
About mathematical terms *13*
HISTORY OF LANGUAGES OF TEACHING MATHEMATICS IN THE ALGERIAN SCHOOL 14
Teaching mathematics after the independence in Algeria 14
Before the Arabisation *14*
After the Arabisation *15*
The dark decade (1990-1999) 15
Teaching mathematics after the reform of 2005 15
The result of this reform 16
CUMMINS THOERY 16
METHODOLOGY 17
RESULTS 19
Responses for the first question 19
Responses for the second question 20
Responses for the third question 20
Responses for the fourth question 20
Responses for the fifth question 21
Responses for the sixth question 21

Responses for the seventh question 22
Responses for the eighth question 22
Responses for the ninth question 22
Responses for the tenth question 22
Responses for the eleventh question 23
Responses for the twelfth question 23
CONCLUSION 23
DISCUSSION 26
REFFERENCES 27

TEACHING MATHEMATICS IN A MULTILINGUAL CONTEXT IN THE UNIVERSITY LEVEL

INTRODUCTION

It is acknowledged within the field of mathematics education that language plays an important role in the learning, teaching, and doing of mathematics. Language is defined to be a system of linguistic rules and texts, and particularly, an array of contexts of use for many kinds of rules and texts (Morgan, 2013, p.1).Mathematics is taught around the world in many different languages, which are different from the mother language. The language of instruction might be a second or a third language. The choice of some countries of what second language or languages to use in schools and universities depends on issues of access, power and dominance (Edmonds-Wathen, Trinick, and Durand-Guerrier, 2016, p.1).In fact; many post-colonial countries keep using the language of the colonizer country as the language of instruction. As a result, there has been a worldwide trend for schooling in English or in another of the world's dominant languages such as French, Russian, Spanish or Chinese (Edmonds-Wathen, & all.2016, p.1).

Language has been a topic of research in mathematics education for a long time (Morgan,2013, p.1), so what role does the language play in the processes of doing mathematics and producing mathematics knowledge? An important issue is the challenge in translating mathematical terms (Durand-Guerrier, Kazima, Libbrecht, Njomgang Ngansop, Salekhova, Tuktamyshov, and Winsløw, 2016, p. 1).

It is claimed that language has a special role in relation to mathematics because the entities of mathematics are not accessible materially; it means we know that subjects of mathematics are abstract, so languages are important to help a student to understand any subject in mathematics.

Mathematics language is different from natural language, a 'set' in mathematics is different from a 'group', while in natural language these words have almost the same meaning, a 'function' in mathematics has particular definition and is different from the concept of function in a natural language, the same is for 'space', 'distance', rational, and many other words used in mathematics. Moreover, as stated before, when we shift from a language to another, many words do not have the same meaning in different languages.

In Algerian university, and since more than a decade, students' linguistic problems regarding learning mathematics in French have increased. Students are

unable to learn mathematics in French because of their weak language mastery of this foreign language. Many voices, among students, teachers and administrators arose to teach mathematics in Arabic, others called to shift to teaching mathematics in the university in English.

In this research, we will investigate how students really deal with learning mathematics in French, by using similarly two other languages, which are Arabic and dialect. We also want to know what students think about the role of language and how do they do to manage their language difficulties. Our particular aim is to find out students' beliefs, thoughts and suggestions about the problem of teaching mathematics in French in the university.

Our work has been completed in nine parts; in the introduction, after having identified the aim of our research, we have presented the situation of teaching mathematics in a multilingual context in many countries. The second part is a literature review; we have aimed to show the difficulties of teaching and learning mathematics in second or third language, regarding some particular mathematical aspects. In the third part, we have briefly presented the history of languages of teaching mathematics in the Algerian school with the different periods. In the fourth part, we have provided information about Cummins theories, one of the important theories about learning process in a second or third language. The part of the methodology is the description of our experiment, which reported in detail, the interviews performed with students. The sixth part contains our results, which are the responses of students for the interviews. The next part is the conclusion, where we have summarized the main ideas emerged in the results. Finally, in the discussion, we have discussed the conclusion regarding the purpose of this study.

Teaching and learning mathematics in a second or a third language in the world

In all over the world, many countries are witnessing increasing language diversity regarding teaching and learning mathematics. It is due to immigration caused by wars or civil wars(like Iraq and Algeria during the nineties to European countries), decreasing economic situation (Latin people to USA, and African and Arabic people to Europe), climate change (African countries to Europe), or global mobility (multinational firms, undergraduates mobility for scholarships,…). But it seems that even before these phenomena, language diversity has always existed in some parts of the world like post-colonial countries (English for India, French and English for many African countries, Portuguese in Macau in China, English in Hong-Kong, English for Ireland) and countries with autochthones and minorities like (Tatar in Russia, Berber (or Amazigh) in Maghreb countries, Catalan in Spain, Indians in Australia). We will present the particular linguistic situation of three countries where teaching and learning mathematics is made by the use of more than one language.

Denmark

As a member of the European Union, Denmark takes part in the student exchange program Erasmus, with about 2,416 outgoing and 6,186 incoming students in the academic year 2009/2012 (numbers which are continuously increasing). The total number of exchange students is about 16,000 (incoming) and 8,000 (outgoing) every year. However, compared to the total number of students in higher education in Denmark (about 200,000), the numbers are not very high. The aim of the government is to facilities access to Danish universities for foreign students, but the principal obstacle for foreign students can be the Danish language, which very few students have had the opportunity to learn. In the last decades, more graduate courses are offered in English – particularly within scientific and technical fields(Durand-Guerrier, Kazima, Libbrecht, Njomgang Ngansop, Salekhova, Tuktamyshov, and Winsløw, 2016, p.8).On one hand, the government thinks it is hard to require short-term exchange students in a mathematics program to learn Danish well enough for academic teaching (instruction) in this language. On the other hand, in mathematics, it has been showed that not all students and teachers are able to switch languages, so it has been decided, in some universities, to go towards offering all texts in English, and at least some of the teaching would be in English at the graduate level. Based on experience, this could work well, but with major obstacles:

- Not all exchange students are fluent in English and in fact many are much less fluent in English than the average Danish student.
- Some immigrant students who have fought to learn a second language (here, Danish) well, (and may have chosen mathematics or other technical subjects in part because they had difficulties with other subjects) may now be asked to learn a third language, which is 'English' (Durand-Guerrier, & al., 2016, p. 8).

France

In France, there is only one official language, French, which is used both in everyday life and for instruction in schools. For historical reasons, French is also an official language in some other countries in Europe (Switzerland),in numerous countries in Africa (like Algeria),in a few countries in the Americas or Oceania (Canada).French might be not the only official second language, in some countries; often English is also presented as a third language. There are also countries, mainly in Northern Africa, where French is not an official language but is still used as the language of instruction in mathematics a certain grade on (e.g., secondary school in Tunisia). For this reason, France has historically attracted university students from a variety of countries. Recently, due to student mobility, the number of countries welcoming foreign students is increasing. Research provides a synthesis of the results of a collective analysis of the UNESCO, OECD, and Eurostat (OED). The results are given for 2007–2008. In France, there were 243,400 foreign tertiary students, representing 11.2

% of the total amount. The increase from 1999 to 2000 is about 75 %. Students, from Africa, are the majority (43.5 %), far more numerous than those from Europe (21.3 %), or from Asia (21.0 %). Students from China constituted in 2007–2008, the second most important group of foreign students in France(Durand-Guerrier, & al., 2016, p.9). These foreign students are studying in France in a great variety of domains (e.g., Social Sciences, Law, Health and Social Sector, Engineering) and particularly mathematics in French, which is a second or a third language for them (Durand-Guerrier, & al., 2016, p. 10).

Cameroon

Cameroon is a country in central Africa with French and English as two official languages. The educational system is divided into two main subsystems: The Francophone subsystem and the Anglophone one. The two subsystems are so different. Any student who started his education in one of those subsystems cannot easily continue studies in the other subsystem, mainly because of the language barrier (difficulties), even though the language of the other subsystem is taught in each subsystem, with a very modest level of proficiency. In higher education institutions, Cameroon has Anglo-Saxon style universities such as the Universities of Bamenda (in the North-West region) and Buéa (in the South-West Region). The other universities are bilingual; courses are given in French and English, depending on the teacher's dominant language (Durand-Guerrier, & al., 2016, p. 7).

LITERATURE REVIEW

Difficulties of teaching and learning mathematics in second or third language

Mathematics is taught in many different languages around the world. In some countries mathematics is taught in only one language, for example in the medium of English in England, but in other countries in two or more languages. However, the fact that a community is multilingual does not imply that every member of the community can speak all the represented languages (Edmonds-Wathen& al. 2016, p.1).The main problem in multilingual educational context is the misunderstanding that can appear as a consequence in the meaning of the words in the languages of instruction and the dialect language, as claimed in (Edmonds-Wathen& al., 2016,):

'We should know that languages express mathematical thinking in diverse ways; these different ways can provide opportunities to enrich the mathematical experience of learners in multilingual contexts. But also, they can introduce a block or misunderstanding between teachers and students and doubt the process of mathematical learning' (cited in Durand-Guerrier, 2016, p.1).

About logical features of language

Quantifiers

According to Dubinsky and Yparaki (2000), in natural language the interpretation of AE and EA (which are for all, there exist and there exist for all) statements are interpreted according to the context. For example, the statement "There exists a mother for each child" (EA) has the same meaning as when we say: "for each child there exists a mother" (AE), due to the context. Which is impossible in mathematics context for example, we should clearly distinguish between "There exist a number greater than all other numbers" (EA), which is false in the set of natural number, and "for every number, there exists a number that is greater" (AE), which is true in this set (Durand-Guerrier, 2016, p.2).
In Tunisia, Chellougui (2009) showed that first year university students consider that EA and AE statement have the same interpretation, even in the mathematical context (Durand-Guerrier, 2016, p.2).

Mathematical negation

Many mathematics teachers consider that in some situations the mistakes made by students are logical in nature. Among these errors there are those relating to the notion of negation of universal statements (Benkilani, 2003, p. 1).
At first glance, the concept of negation could appear as a very simple one, met and used early by children. This simplicity, however, holds only for singular statements, such as "π is a rational number", whose negation is "π is not a rational number". In fact, as soon as quantifiers appear, negation becomes more complex, for example: "Some *A* is *B*" (1) and "Some *A* is not *B*" (2). As already realized by Aristotle, while sentence (1) is affirmative and sentence (2) negative, despite, we see the 'not' of negation in the sentence (1), but there is no opposition between the two(Durand-Guerrier, & al., 2016, p. 14).

The structure of negation in English is expressed like in natural language; it is formed by do (in the corresponding tense: present or past) + not +verb (example: I do not eat meat) (Azrou, 2020, p.4).
In French, a standard way to build negation is the use of the locution "ne … pas". For example: "toutes les boules sont rouge"its negation is: "toutes les boules ne sont pas rouges."(Durand-Guerrier, & al, 2016, p. 14). When we talk about the negation, we can see we negate only the verb with 'ne' and 'pas'. Another example, in the domain of natural numbers "to be odd" is equivalent to "not to be even". Let us consider now the following statement: "Tous les diviseurs de 12 sont pairs" ("All the divisors of 12 are even"), which is false. Its standard negation in French "Tous les diviseurs de 12 ne sont pas pairs" ("All the divisors of 12 are not even") is true. Then, if we change "ne sont pas pairs" ("are not even") in "sont impairs" ("are odd"), we will have: "Tous les diviseurs de 12 sont impairs " ("All divisors of 12 are odd"), which is false (Durand-Guerrier, 2016, p.4), here we can see that "ne sont pas pairs" is the same "sont

impairs" in the meaning of the writing but is totally different in mathematical concepts, like in the second example. So, the negation taught in the French class (in French language course) transforms a universal sentence into a universal sentence by only making the negation bear on the predicate i.e. the negation of (∀x, Fx) is (∀x, no Fx), but the logical negation transforms an affirmative universal proposition into a negative existential proposition; (∀x, Fx) its logical negation is (∃x, no Fx) (Benkilani, 2003, p.1).
In classical Arabic, the situation is almost the same; we say in mathematics 'nefye P' for 'negation of P or non P and in the natural language, we negate verbs by using only 'la' before the verb (la+ verb), 'la' is also the opposite of 'yes' in Arabic. I also negate by using 'laysa' which comes before a noun while 'la' comes before a verb. But in dialect, it is a little bit different: when we negate, we use a prefix 'ma' attached to the verb followed by a suffix 'esh' (ma+ verb +esh: ma+ eat +esh means 'I do not eat'), while the word for 'no' is 'lala' or 'la' (according to the different regions in Algeria) (Azrou, 2020, p.4).

Conditional structure

We present here an overview of the conditional form in Arabic documented in details by Azrou (2020).
The conditional structure 'if … then', which plays an important role in mathematics, is presented in different ways in the three languages (Arabic, French and Algerian dialect).In mathematics, the only Arabic form taught in schools (from upper secondary) is expressed 'idha…idhen' ('dh' is pronounced as 'th' in then), if we consider this example: if a function is derivable, then it is continuous. However, this structure (using idha...idhen...) is not regularly used in Arabic and does not sound good.
In Arabic, like in other languages, the conditional sentence is composed by two parts: the protasis, i.e. the sentence of the condition (with the verb of the condition), as 'the function is derivable' in the previous example, and the apodosis, i.e. the consequence of the condition, as 'the function is continuous' in the previous example (with the verb of the consequence). The terms put in the place of 'if' and 'then' are many and are not used two by two necessarily (like 'if' is only used with 'then').
Focus in Arabic is on the conditional terms put at the beginning, there are many and fall into two groups: those that change the ending of the first verb and those that do not. Among those that belong to the first group, there are seven (inna, idhma, mahma, kayfama, haythouma, ayy). The second group is composed by (idha, law, lawla).
- 'ithen' (then in English) is rarely used, there is another word used more often, which is 'fa', or we use nothing, so the conditional structure is often expressed as: 'idha…fa'…', or 'idha...nothing...'
- There is another word used for 'if' which is 'in' that would give 'in…fa…'
- There is another word used for 'if' which is 'law' that would give 'law…fa…'

- There is another word for ‘if’ which is ‘lamma’ as the synonym of indama (when).

In French (even in English), the way to express the conditional situation is different compared in Arabic let see this example “if I have a car I will visit you soon, in Arabic we say: Law + verb +car la + verb +you, ‘in ...la...’, or ‘in ...sa...’.

To express a condition that has never been realized, in Arabic we use ‘law…la…’, with the past tense for the verb of the first sentence and a particular mode for the second sentence we can see that there is no correspondence with what happens in the same situations expressed by the conditional ways in French (also in English) to express, possible, probable or impossible situations, by changing the tense and the mode of the first verb and the second verb(Azrou, 2020, p.2).

Even though, in high school mathematics, we have introduced one structure to express a condition, which is ‘idha…ithen…’ it is not sure that students would consider other conditional sentences (formed with other terms, cited above) as being the same as this one. But more importantly, students might not be able to translate Arabic sentences with different conditional terms (other than ‘idha’ and ‘idhen’) to French sentences with ‘si… alors…’ (if…then…) and when they translate from French the conditional sentence (si...alors...) to Arabic, students might not consider the multiple Arabic terms to express the condition other than ‘idha...idhen...’, particularly in the oral form (Azrou, 2020, p.3).

In the dialect, the words are different, the analogous of ‘if’ is ‘ki’ and there is no words for ‘then’ in the present; while for a condition that has not been satisfied, the analogous for if is ‘lukane’ and no words for ‘then’ or we use ‘lukane’. So we say: ‘lukane… (Nothing)…’, or ‘lukane…lukane…’ In this structure, the verbs for the first and the second sentence are always in the past.

Again here, we wonder if students, when thinking in dialect or listening their teacher who speaks in dialect would translate their conditional structure to the French one with ‘si…alors’ and when they translate from French (for instance when they read a textbook) No need to mention how it is confusing and complicated when a student thinks in dialect speaks in Arabic and translates into French to write a text, or when a student listens in dialect, thinks in Arabic and writes in French. The problem is how students would deal with other conditional structures in Arabic using other terms than ‘idha’ and ‘idhen’, whose translation in French is ‘si’ and ‘alors’ (Azrou, 2020, p.3).

Mathematical implication

About mathematical implication, we have: if you come with me I will give you some cookies, it means that if you do not come with me you will not get the cookies, which means that an implication in a natural language as P implies Q, has the same meaning as (not P) implies (not Q).But in mathematics, the implication is totally different, and its situation in the natural language does not

hold in mathematics; in other words, (P implies Q) has the meaning of ((¬P) ∨ Q) and (¬Q implies ¬P), while (¬P implies ¬Q) does not hold in mathematics. Students might have often, difficulties understanding many mathematical statements expressed this way, if they do not know this important distinction.

About particular mathematical terms and language

About mathematical terms; when students enter the university, they already master the mathematical terms used in the past three years, like: limit, function, continuous, derivative, set, line, equation, inequation, etc. When students start to learn mathematics in French at the university, they try to translate French into Arabic and would feel secure when they are given a mathematical term in French, for which they already know its translation into Arabic (Azrou, 2020, p.5). The problem happens when they begin to learn new mathematical terms related to concepts and definitions of undergraduate mathematics not already met in high school (example: group, intersection, injection, morphism, restriction, inferior bound, etc). In fact, this is the case with students all over the world, but the difference is that with any other student (Italian, English, French…); a new mathematical term is accepted as such. With our students, who base their learning on translation from French into Arabic, when they encounter a new mathematical term in French, whose translation in Arabic does not correspond to a known term (or not known by the teacher), they panic, feel stuck and continue to look for a similar word in Arabic in order to be able to understand the mathematical meaning(Azrou, 2020, p.5).

The first problem to face, in a multilingual context, is about translating or language shift. In fact, when translating mathematical terms from a language to another, it is easy to see that there are lexical differences across languages. For example, when considering everyday language, the English word ‘field’ could be translated into French as ‘domaine’, ‘champs’ or ‘terrain’, but none of these translations are valid when considering the use of the term ‘field’ in algebra, where the correct French translation is ‘corps’. More delicate examples of these differences are pairs of expressions in two languages that look or sound similar, but differ significantly in meaning. Such differences may have deeper consequences for understanding mathematical texts. For example, in French, ‘nombre décimal’ has a finite number of non-zero digits on the right hand side of the decimal separator like: ‘1/2=0.5’", while in English, a ‘decimal number’ can have infinitely many non-zero digits like:

"1/3=0.333333…"(Durand-Guerrier& al., 2016, p.2)

Mathematical language in Tatar has its own properties especially at the lexical and grammatical level, regarding the special terminology, phraseology, and syntax, in terms of lexical and grammatical structure, and in the genre of mathematical texts. For example, there are general syntactic differences between word order in sentences in Russian and Tatar; the main difference is in the fact

that the verb in a Tatar sentence is always written at the End. Another example more specific to mathematics is the difference in the names of fractions in Russian and Tatar. In Russian, the value of the numerator is spoken before the denominator, while in Tatar it is the opposite. For example, the fraction 1/2 in Russian is read as "odnavtoraia" (1 over 2), and in Tatar, "ikedenber" (2 under 1)(Durand-Guerrier& al., 2016, p.17).

In Algeria, the same problem about pair of mathematical words in Arabic and French having different meanings exists also. For example, the Arabic mathematical term for a rational number is 'a talking number' (عدد ناطق), in French it is said 'nombre rationnel' which would be translated to Arabic as ('عدد عقلاني'), which means a number with some common sense. Likewise, the irrational number is said in Arabic as a deaf number (opposite to talking number) (عدد اصم), while in French is it said 'nombre irrational' the same as English.
Another example is the term integral ('intégrale'in French) or 'التكامل' in this word the French verb 'intégrer' is inserting something into another, by translating, a student might understand that meaning. In the Arabic dictionary, we cannot find the word'التكامل', but we can find the word 'intégralement' 'الكامل', which means something that is complete and whole. Students, who translate regularly, might confuse or adopt a term in another language, whose meaning is wrong, and this could impede students from understanding mathematics.

HISTORY OF LANGUAGES OF TEACHING MATHEMATICS IN THE ALGERIAN SCHOOL

Teaching mathematics after the independence in Algeria

Before the Arabisation

On July 5th 1962, Algeria became independent. At that time, it was not possible to meet the needs of teachers able not only to teach the Arabic language, but above all to teach others disciplines as mathematics in this language. Faced with this lack of supervision, it was necessary to call on the former colonial power. More than half of the 15,000 French cooperative put at the disposition of the new state are teachers who originally came back to Algeria for the start of the 1962 school year (Exposition, 2018, p.16), other cooperating teachers came from brotherly Arab countries, and hundreds of Egyptian, Syrian and Iraqi teachers have supported Algeria's huge education effort. Mathematics was taught in French in all school levels and in the university. With the project of Arabization of the school, undertaken in 1965, more classes were taught in

Arabic, and so, children began to learn mathematics, in Arabic in the school and later in high school. This project did not include the university, regarding technical sciences (Taleb Ibrahimi, 2015, p.54).

After the Arabisation

The teaching of mathematics in Algeria has shifted from French to classical Arabic (oral, writing and mathematical symbols) for the three school levels (primary, middle and secondary school) since the 70's (Azrou, 2019, p.2). While in the university level, teaching mathematics was still performed in French language, as for other technical sciences disciplines.In 1976, the minister of education instructed the schools to use Arabic as a unique language of schooling with teaching French as a foreign language, from the third grade. The process of total arabization is completed for the three school levels in 1986.At the start of the 1988-1989 school years, primary, middle and secondary education was completely arabized (Taleb Ibrahimi, 2015, p.56).

The dark decade (1990-1999)

During the civil war (in the nineties), teachers of French were threatened and many of them have been killed, because it was thought that only Arabic language should be taught in school in Algeria. As a result, many students lacked teachers of French in all school levels; and missed several years of learning French; as a consequence, students grew and passed through different school levels with a weak level of French language, till reaching the university. This situation affected the learning of mathematics at the university, which has always been in French language. Students arriving to the university, in technical sciences, with weak French mastery, had big difficulties to attend courses where teachers speak, write and read mathematics in French. This gave rise to the next reform about the language of teaching mathematics in the school.

Teaching mathematics after the reform of 2005

The school reform, which officially started in 2005 aimed to make the transition to university mathematics less difficult (Azrou, 2019, p.2), teaching of mathematics is done in two languages: the Arabic language and French, at all school levels. The writing of mathematical formulas, symbols, and equations was in French (from left to right), while Arabic is used for verbal expressions and mathematical terms (from right to left), for example:

Find the area of a rectangle ABCD of length 20 m and side width 15 m, is written, as follows going from right to left and from left to right many times in the same sentence.

-اوجد مساحة المستطيل ABCD طول ضلعه15m و عرضه20m

The result of this reform

The reform caused many difficulties for children and teachers in the school. But more importantly, it could not help, unfortunately, students arriving to the university to learn mathematics in French without difficulties. Their weak language level in French impeded them to have courses where they read, hear and write in French.

CUMMINS THOERY

We will consider the linguistic theories of Cummins (Cummins, 1981a) to explain better the process of learning two or more languages. Cummins's language acquisition theories make the distinction between two differing kinds of languages proficiency:

Basic interpersonal communicative skills (BICS) are the surface skills of listening and speaking a language. They are typically acquired quickly, by many students learning in a second language (Cummins, 1981).

Cognitive academic language proficiency (CALP*)* is a level of second language that is required to cope with the academic demands placed upon a student in the various subjects in the school (Cummins, 1981).

Cummins states (Cummins, 1981a) that many children develop native speaker fluency (BICS) within two years while academic language (CALP) can take from four to seven years depending of many variables such us age and time of arrival to school. These two types of second language skills help teachers and educators to get a better understanding of second language ability and draw their attention to the different challenges and difficulties that second language learners meet.

Later, Cummins included two distinct types of communication depending on the context in which the communication occurs, so he distinguishes between two kinds of context (Cummins, 1994, p. 1):

1- Context-embedded communication: where students can look at illustration of what has been said about or ask questions to confirm understanding, it helps making the information comprehensible. For example: objects as when a teacher explained for children in primary how to make an addition or substation, or by gestures.
2- Context-reduced communication: provides fewer communicative clues, which means there are not many ways to explain and no sources of help than the language itself. Like a phone conversation, a note left on a refrigerator.

According to Cummins, language can be cognitively undemanding or cognitively demanding, the first one requires a minimal amount of abstract or critical thinking for example a conversation in playground or, a simple Yes/No

question. The second one requires a learner to analyze information quickly and often contains abstract concepts such as a math lesson or a multiple-choice test.
Common underlying proficiency(Cummins, 1994, p. 2):
Cummins (2000) believes that in the course of learning one language a child gets a set of skills and implicit meta-linguistic knowledge that can be drawn upon when working in another language. This common underlying proficient (CUP) provides the base for the development of both the first language and the second language. This theory also serves to explain why it becomes easier to learn additional languages. So any expansion of cup that takes place in one language will have beneficial effect on the other language(s), for example "if a child already understands the concepts" of 'justice' or 'honesty' in her own languages, all she has to do is acquire the label for these terms in English or her second language she has a far more difficult task, however, if she has to acquire both the label and the concept in her second language.
We can summarize Cummins's theories as follows:
- Second language learners become proficient in CALP years after being proficient in BICS.
- The ease of understanding language depends on the context in which it is used; context embedded is easier to understand than context reduced communication.
- As Cummins (2000) states, by means of CUP, if a child can understand the concept of any word in her own language, it helps this child to make input in the other language comprehensible. That is why it is very important to encourage students to continue their native language development. (The example of 'justice' and 'honesty' in the meaning of CUP)

METHODOLOGY

Our experimentation consisted in making interviews with students of different levels due to the fact that interviews could reveal clearly how students deal with learning mathematics using different languages. The period of interviews lasted from April to July 2020 [1].Semi-structured interviews have been performed with students who have accepted to collaborate with us. Students have been called by phone, the interviews lasted from 20 to 40 minutes with each student, and the responses have been recorded and then transcribed. To ask the questions, we have used either English, for students who have a good mastery of English language or Arabic and Algerian dialect for students who have low mastery of English language. We had eleven students, ten girls and one boy: 2 L1 students, 2 L3 students, 2 M1 students (one girl and one boy), and 5 M2 students. Most of interviewed students were from Medea, only three students (girls) were from the University of Msila (in the east of Algeria).

[1]The first part of this period was the lock down in Algeria caused by the pandemic of corona virus.

In the following, we present the questions of the interviews with our motivation for each question.

Q1. Do you think that learning mathematics at the university in a new language different from the language students have used before may create difficulties for students? If so, what kind of difficulties?
We aimed to find out the difficulties of shifting languages in learning mathematics at the university.

Q2. Do you think the situation would change if we could use English instead of French?
This question would reveal whether students' linguistic difficulties are linked only to French as a language, and thus the situation would be better with another foreign language as English.

Q3. Does the problem of language concern only first year students or does it persist longer with higher levels (like master students)?(Only for master students).
This question meant to be asked only for master students who have a distance from changing the language of instruction; they might tell us from their personal experience how long the linguistic difficulties lasted.

Q4. Many students ask their teachers to explain and speak also in Arabic or in dialect; do you think this is helpful to them?
We know that by using three languages in learning mathematics, it could be problematic to students, however, the responses of students to this question would show whether it is the case and in which way it could be helpful to students, according to them.

Q5. Many students ask their teachers to translate from French to Arabic many mathematical sentences (statements and verbal expressions); do you think this is helpful?
We would like to know, by this question, whether the mathematical sentences in Arabic are more understandable to students than in French.

Q6. Which language should teachers use to explain mathematics during teaching (lectures and exercises sessions) to help students?
Students' responses for this question is their choice for the best language for their learning, we also expect the how and why behind it.

Q7. In which language do students think during lectures?

As we have doubts that students think in French, we wanted to know exactly, from students, in which language do they think, because we think that the language of the thoughts is the dominant language of learning.

Q8.Which language do students use when they work together to solve exercises?
It would be interesting to find out how students use the three languages (Arabic, dialect and French) when they are solving exercises with each other.

Q9. Which language do students use during the exams (for thinking, solving tasks and writing)?
We would like to know how students do when they work alone, during solving the exercises in the exams. As the exam text is written in French, and as students use other languages (Arabic and dialect), it is very important to identify in which language or languages, the personal process of problem solving occur.

Q10. Do you think that the language used for mathematics at the university could be an important reason of failure for some students?
By this question, students would tell us exactly how much is the failure caused by the language difficulties.In particular, could we consider French language as a factor of failure in learning mathematics at the university?

Q11. Taking into account that French is the language for mathematics at the university, why do several students not improve their learning of French?
This question aims to know why do students not overcome the problem of French mastery even being aware about their weak French level.

Q12. What would be the solution to overcome students' difficulties in mathematics at the university related to the language used for teaching?
Responses to this question would show us how students imagine the solution to their linguistic difficulties, but also how do they interpret their problems regarding the language of teaching mathematics in the university.

RESULTS

Responses for the first question

Q1. Do you think that learning mathematics at the university in a new language different from the language students have used before may create difficulties for students? If so, what kind of difficulties?
R1. All interviewed students (of the four levels L1,L3,M1,M2) said that it is hard for them to study mathematics in a new language different from the language students have used before: *'yes, it creates big difficulties, because when we were in high school we studied mathematics in Arabic, it was good for us even with some mathematical difficulties, we understood very well, but when*

we studied mathematics at the university in French, we could not follow up with teachers, it is hard especially in the first and the second years'.

Responses for the second question

Q2. Do you think the situation would change if we could use English instead of French?

R2.L1. The two students said the situation might change: *'yes, there are a few students who know English well it will be ok for them, but for the majority, French or English does not make the difference, the difficulties will stay the same because students have a problem of mastery language.*

L3. The two students said that there is not a big difference between English and French: *'yes, I think the level of students would be better than now, because all mathematics textbooks are in English when searching in internet, so it might be good to change the language to English, but not necessary because there would be not a big difference'.*

M1. The two students said that will be a good thing: '*yes, it is very good for me to study mathematics in English'.*

M2. Three students said that it is easier than French: *'yes, English is easier than French, besides, when we do research, mathematics textbooks and papers are in English so they should teach us mathematics in English, to help when doing research'.*

One student of M2 said it is the same: *'it is the same thing with French or English the problem would be the same, because we do not have a good base in languages, so we will not understand better, except if we continue to study mathematics with the language that we used before which is Arabic, this might help us'.*

The two other students said it would be harder for them if they change the language:*' no, it is harder, we have been used to French language, and I think that the problem is not French or English, it is in the mathematical concepts'.*

Responses for the third question

Q3. Does the problem of language concern only first year students or does it persist longer with higher levels (like master students)?(Only for master students).

R3. The twelve students said yes, the problem of language appears only in the first years*: 'yes, after the first or even the second year, if students do their best to learn French language at home or with internet, it will be easier to overcome the problem of French, students would understand better their teachers and they would be familiar with some mathematical terms, like the meaning of these words: theorems, proofs, definitions, proposition'.*

Responses for the fourth question

Q4. Many students ask their teachers to explain and speak also in Arabic or in dialect; do you think this is helpful to them?

R4. The students of L1, L3, M1, and four students of M2 said it depends on what the students ask the teachers to explain: sometimes it is helpful for students but not all thing just verbal expression. But sometimes it is not helpful for some mathematical concepts: *'no, it is not helpful for the mathematical concepts because some of them, for example, in topology course as: (المتراصة الفضاءات= espaces compactes, الفضاء الشعاعي=espace métrique). If teachers explain these mathematical concepts in Arabic, it would be harder to link the meaning of their definition in Arabic with the meaning in French, so it is not easy to remember them, maybe because they are new concepts for the students, I think teachers give these concepts in French and explain them to students by giving examples should or drawings but not explaining in Arabic because students should be familiar with these concepts in French'*. Except one students of M2, said it is helpful if a teacher explains in Arabic but just what students have studied before (high school) and the new mathematical concepts for the students but not verbal expressions: '*yes, it is helpful if teachers explain in Arabic in the first two years because students have already an idea before about some mathematical concepts, so it would be easier for students to remember them and this would help for more understanding'*.

Another student of M2 said it is helpful for them: *'yes, students can understand better when teachers explain some words or some verbal expressions which look hard for students in dialect, but not to Arabic because we are not also fluent in Arabic. Teachers can explain them to Arabic, just some mathematical terms about some concepts, I think this would be helpful for students in the first years, and only for the new concepts, which students have not seen before'.*

Responses for the fifth question

Q5. Many students ask their teachers to translate from French to Arabic many mathematical sentences; do you think this is helpful?

R5. All students said it would be helpful but not translating all: '*yes, it is helpful for students, especially for those who are good at math but have a problem with French language, I mean have a weak mastery of French, but teachers should translate only a few words when students are new, like a sentence which seems ambiguous but not all what teachers write should be translated, in order to encourage students to be familiar with some mathematical concepts in French'.*

Responses for the sixth question

Q6. Which language should teachers use to explain mathematics during teaching (lectures and exercises sessions) to help students?

R6. Eleven students said teachers should speak all the three languages (dialect, French and Arabic) to facilitate understanding mathematical concepts: '*teachers should speak dialect, Arabic and French when they are explaining, but only in the first years, after the first years students will understand some new words in mathematics used by the teachers and he will be familiar to these words , the teacher is not obliged to explain the definitions of mathematical concepts that*

the students have learned before in high school, he just can translate these mathematical concepts in Arabic'. Except one student of L1who said teachers should speak only in French to make students be familiar to mathematics in French: '*teachers should speak only in French, because students should be familiar with mathematics concepts in French language'.*

Responses for the seventh question

Q7. In which language do students think during lectures?

R7. All students of the four levels (L1, L3, M1, and M2) said that they think in Arabic for the definitions of some concepts they have learned before and dialect for the words that are in French, which are easy and are used in their daily life as "alors, donc, c'est-a-dire...": '*when we are in the classroom, we translate to Arabic the definitions of the concepts we have learned before to better understanding and we use the dialect to translate simple verbal expressions and words like: 'alors, donc, et, mais...'.*

Responses for the eighth question

Q8. Which language do students use when they work together to solve exercises?

R8. All students said they explain to each other in dialect, but some definitions of mathematical concepts like: integration, derivation, limit, they explain them like they have learned in French: *'when we are explaining to each other for solving exercises, we use dialect and Arabic, we translate to dialect and Arabic the new mathematical concepts, words and definitions, but for simple questions with known mathematical concepts, like sequence, derivation, limit, Riemann integral, we do not translate, we try to remember their definitions in French, and write directly the solution in French'.*

Responses for the ninth question

Q9. Which language do students use during the exams (for thinking, solving tasks and writing)?

R9. Elevenof students said they would translate only when it is hard to them to understand: *'if it is hard to understand the meaning or the goal of the exercises in the exam, I try to translate to dialect or Arabic to remember the definitions I have learned before, whether it was in previous years at the university or in the same year, if it seems easy I would solve the exercise in French'*. Except one student of M2 who said that students use Arabic and French in thinking, but only French to write the solution: '*In exams they use Arabic mixed with French terms in thinking, and French in writing'.*

Responses for the tenth question

Q10. Do you think that language used for mathematics at the university could be an important reason of failure for some students?

R10. Nine of students said yes it could*, 'yes, the language can be an obstacle for some of students who really can't learn the languages and have a very bad base*

before'. The rest of students*, 'no, I think that the languages never can be a reason of failure for students, because they can just know and memorize some concepts as: 'sequence, function, continuity, convergence', symbols in mathematics in French by solving exercises and revising the courses regularly in each year, if they are good in mathematics they can success even they are not fluent in languages'.*

Responses for the eleventh question

Q11. Taking into account that French is the language for mathematics at the university, why do several students not improve their learning of French?

R11. The two students of L1 and one student of M2 said because they do not have the basic level of French language and do not know it could be an obstacle for their future success: *'perhaps students do not give importance or they do not know the importance of languages at the university especially French, they think that if they are good in mathematics it is enough, and unfortunately they will be shocked during the first years'*. The remaining nine students said that it is because of the absence of support from their parents: *'when our parents have remarked our bad marks in languages 'French or English' in primary and middle school, they did not help us to improve our level and when we grew up and discovered our bad levelit was late to catch up in high school and at the university even we made efforts to learn French'*.

Responses for the twelfth question

Q12. What would be the solution to overcome students' difficulties in mathematics at the university related to the language used for teaching?

R12. The Students ofL1, L3, M2said the solution is not to learn the language in the university they should do their possible to learn it at home or before in the school: '*students should acquire the base for a language during primary, middle, and high school levels, they cannot learn it at the university because it would be too late'.*

M1. One student proposed removing some unnecessary courses in the first year like physics, and replace them with a French language course, which students need more: '*they should teach students a course about the language of instruction according to what they need to improve their level in mathematics courses especially in the first and the second year, but with language teachers who have a good level'.*

Another student of M1 said: 'we should keep studying mathematics in Arabic as we have learned before, because some of my friends who study in ENS have no problem with mathematics as we have'.

CONCLUSION

Responses of students for the first question about the change in learning mathematics in a new language at the university revealed that all of them

recognized having big difficulties when shifting the language from Arabic to French, especially in the first two years.
Answers of students for the second question about the changing of the linguistic situation at the university and whether it would be possible to use English instead of French showed that some students (of L1,one student of M2 and students of L3) recognized that even if English is useful, the change to English would change very little regarding students' linguistic difficulties because the problem, according to them, is about the language mastery of students, which plays a very important role to learn mathematics at the university and not which language to use (be it French or English).Two students of M1thought that English could be better than French. Few students of M2 (two students) who said to be good in English and more aware about research feel the necessity to learn mathematics in English, in the university to be able to deal with papers and mathematics textbooks and all the information found in the literature, mostly written in English. For the three remaining students of M2, the difficulties are more about mathematical concepts and not related to the language of instruction.

About the problem of language whether it concerns only first-year students or does it persist longer with students of higher levels (like master students), according to all master students, the problem about French language appears only during the two first years at the university, and later, students would be familiar with the mathematical concepts, mathematical terms, and some of the used expressions in French so they would not have a problem later.

Concerning the fourth question about whether it is helpful for students when teachers explain the mathematical sentences in Arabic or dialect, most students suggested to not make it for the mathematical concepts which are new for students, because it would not help students to be familiar with them; according to them, mathematical concepts, which are present regularly, should be used and kept in French. The exception was for the two students of master2, who evoked explaining mathematical terms in Arabic and French verbal expressions in dialect. The other preferred to explain in dialect, because according to him Arabic is not mastered, by students.

Concerning whether it would be helpful for students when they ask their teachers to translate mathematical sentences from French to Arabic in the fifth question, all students revealed that teachers should help students to facilitate the understanding by translating just in the first years, but not all what teachers write on the blackboard just ambiguous sentences, as mathematical concepts and verbal French expressions.

About the sixth question that is related to which language should teachers use to explain mathematics during teaching (lectures and exercises sessions) to help

students, the majority of students declared that teachers could use all the three languages (dialect, French, Arabic) to help students only in the first years, the exception were for two students, one of them of M1 acknowledged teaching in English is better because of all mathematics knowledge on the internet in English, the other (of L1) preferred using French when the teacher explained the courses.

For the seventh question, which is about the language used by students for thinking during lectures, all students of L1, L3, M1, and M2 declared that they use all their known languages (dialect, Arabic, French) to understand their courses in mathematics. They translate the verbal expressions, definitions and the meaning of mathematical concepts, already studied in high school, to Arabic or dialect if it seems hard in French, but if it seems easy for them as some mathematical concepts or definitions which they studied them at university and they understood them well, they keep and deal with in French.

Responses of students to the eighth question, about which language do students use when they work together to solve exercises, revealed that all of them prefer using the three languages all together (dialect, Arabic, French).

As concerns the ninth question about the language that students use during the exams (for thinking, solving tasks and writing), according to most students they use dialect and Arabic if it is necessary to translate, and French to solve questions that seems easy. The exception was for one student of M2, who always translates to Arabic to be able to understand mathematics sentences whatever the difficulty is, using French only for writing.

Regarding the tenth question about whether the language used for mathematics at the university (French) could be an important reason of failure for some students, most of students revealed that the language can be an obstacle due to the weakness mastery of French and the absence of means to be able to continue studying it. The exception was for two students of L3who acknowledged that anyone could succeed if he/she is good at mathematics, solving many exercises and revising regularly their courses, they added that it is enough to memorize some verbal expressions and mathematical terms, which are regularly used.

By responses to the eleventh question, the main reason, which makes several students not improving their mastery of French, is their weak mastery level, which requires starting all over. Students declared that their teaching of French language, in the three school levels, has always been weak and so were their results, but they never knew, before, that French language could be later, in the university, so decisive and necessary and become a big obstacle for their future education.

Students' responses for the twelfth question related to the solutions that would be suitable to overcome linguistic students' difficulties, all students of L1, L3, and M2 think that learning French in the school correctly till the mastery level before coming to the university is the best solution. Two students of M1 had a different idea; one of them thought that teaching French at the university is possible by removing the unnecessary courses, which have no relation with mathematics(like physics).The other student preferred studying mathematics in Arabic, similarly as students who study in Arabic in ENS[2].

DISCUSSION

All students showed their awareness about the difficulties of language in learning mathematics at university, related to using French as the language of instruction, after having learned mathematics in Arabic (with French written formulas) in school. Many students are also aware that the language mastery of students is more important than the language to use, be it French or English. This sustains that the problem of teaching mathematics in French at the university is not about the French language but about the mastery of French language, by students, which seems to be very weak. Only few students who feel the necessity of English used in research, particularly with scientific papers and mathematics textbooks would like learning mathematics in English. With a weak French level mastery, students acknowledge overcoming their French language difficulties to be hard, even over time; that is why they prefer to be only familiar with French verbal expressions and some basic mathematical concepts, which are used regularly in mathematics courses. With their incapacity to use French fluently, students are constrained to use similarly all their known languages: Arabic and dialect. They also ask their teachers to use them, with some moderation, by translating only the mathematical terms, which they already know and keep those, which are new to them, in French.

French is avoided by students when they work together, and during the exams. It is used only for mathematics formula, as it has always been so, before in the school.

An important issue emerged in the responses of students related to the role of language in their practices to learn mathematics in French, they seem to underestimate it, but worse, they completely ignore the role of language in the learning process, indicated by the following facts:

Students believe mathematics is reduced to formulas, for problem solving they need to memorize procedures and some particular methods, as they were used before in the school, which would minimalize the use of language. This shows how weak and superficial they learning of mathematics are.

[2]A faculty in Algiers for training teachers, where the language of teaching is Arabic

Students declared that the problems of language disappear after the two first years as they become familiar with most mathematical terms and verbal expressions. We were expecting them to say they have mastered the French language, after five years; unfortunately, being familiar does not mean that someone could use a language at the academic level. In fact, according to Cummins' theories, it takes five to seven years to someone to master a second language academically, which means that our master students would have mastered French academically after five years, if they had a good communicational French level during the first years, at the beginning. But, students did not reach such mastery level, and cannot reach it as they content themselves by being familiar with some mathematical terms and verbal expressions, which are used regularly. What about new mathematical terms and new verbal expressions? What about thinking and dealing with ideas, which would build their mathematical reasoning in the process of learning mathematics?

Students think they would do better in mathematics, when they use three languages, but we think it is completely the opposite. The fact that students use a language to talk, a language to think and a language to write is weakening their abilities, because none of these languages is mastered at the academic level. But also, the fact that these three languages have different linguistic structures (Azrou, 2020), does not help students to think, to express and to write the same thing (students do not write in the language they use to think, do not think in the language they use to talk and to not talk in the language they use to write) and this seems to be problematic.

To overcome this problem related to linguistic difficulties, students sustain the fact they should learn French, very well in the school, before arriving to the university. Moreover, students would like to be informed in the school about the importance to learn French fluently to be used in the university in technical sciences. Only one student, suggested to study in Arabic, this shows students are open to study in foreign languages, even acknowledging their weak linguistic level mastery.

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