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Abstract

The objective of this study is to provide an understanding of the Standard Model, a theoretical framework that characterizes the fundamental particles and forces in the universe. By adding extremely heavy particles, which results in a small neutrino mass, the Seesaw Mechanism provides an explanation for the modest mass of neutrinos. Without changing the Standard Model's basic structure, this hypothesis gives it a new dimension and improves its capacity to explain neutrino features.

Résumé

L'objectif de notre étude est d'offrir une vision du modèle standard, qui représente un cadre théorique qui décrit les forces et les particules essentielles de l'univers. La petite masse des neutrinos est expliquée par la théorie de Le mécanisme de Seesaw qui intègre des particules très lourdes, ce qui entraîne une masse réduite pour les neutrinos. Une nouvelle dimension est apportée au modèle standard par cette théorie, ce qui lui permet d'expliquer les propriétés des neutrinos sans modifier sa structure fondamentale.

ملخص

تهدف دراستنا من خلال هذا العمل إلى تقديم مفهوم حول النموذج المعياري الذي يتمثل في إطار نظري يصف القوى والجسيمات الأساسية في الكون. تقدم نظرية التأرجح تفسيرًا لصغر كتلة النيوترينو من خلال دمج جسيمات ثقيلة جدًا، مما يؤدي إلى كتلة صغيرة للنيوترينو. تضيف هذه النظرية بعدًا جديدًا للنموذج المعياري، مما يوسع من قدرته على تفسير خصائص النيوترينو دون تغيير هيكله الأساسي .

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General Introduction

The Standard Model of particle physics explains the fundamental particles and their interactions, excluding gravity. It categorizes particles into fermions, which make up matter, and bosons, which act as force carriers for electromagnetic, weak, and strong interactions. Despite its success in numerous experiments, the model is incomplete, as it does not account for gravity, dark matter, or dark energy.

In particle physics, cross sections quantify the probability that particles will interact, offering vital details for forecasts based on experiments. Radiative corrections help theoretical predictions better fit actual data by taking into account the effects of electromagnetic interactions, such as photon emission and absorption. In the field of particle physics, they serve as the cornerstone for comprehending particle interactions and guaranteeing the coherence between theory and experiment.

In the first chapter, we will review the Standard Model,focusing on the elementary particles, fundamental interactions, and feynman rules .

The second chapter will be devoted specifically to the mathematical formulation of quantum field theories, including the fundamental concepts used in calculations: the Cross Section, Dyson serier, divergence-problem of infinity, renormalisation, Radiative Corrections (One-Loop Corrections).

In the third chapter, we delve into the fascinating phenomena of neutrino oscillations and the longstanding enigma of neutrino masses. Within this context, we thoroughly examine the Higgs mechanism, which plays a pivotal role in endowing elementary particles with mass. Furthermore, we explore an alternative solution to the mystery of neutrino masses known as the seesaw mechanism.

Chapter 1

The Standard Model

1.1 Introduction

In the field of particle physics, the Standard Model is one of the most effective and complete theories. It was created in the 20th century by physicists working together to develop a fundamental understanding of the elementary particles and fundamental interactions.

The Standard Model categorizes elementary particles into two groups: quarks and leptons. Quarks, combining to form protons and neutrons, and leptons, including electrons, muons, and tau particles, serve as the building blocks of matter. These particles interact through three fundamental interactins: the electromagnetic force, the weak nuclear force, and the strong nuclear force.

The electromagnetic force is mediated by photons, while the weak nuclear force involves W and Z bosons. Gluons, on the other hand, are responsible for mediating the strong nuclear force that binds quarks within protons and neutrons. Additionally, the Higgs boson, discovered in 2012 [1, 2], plays a crucial role in providing mass to particles, completing the particle content of the Standard Model.

The success of the Standard Model is evident in its accurate predictions of experimental outcomes in particle physics. From the precise measurements at high-energy colliders to the delicate observations in particle detectors, the model has solidifying its status as the cornerstone of our understanding of the subatomic world.

However, the Standard Model is not without its limitations. It does not incorporate gravity, and it leaves unanswered questions about dark matter and dark energy, suggesting the possibility of physics beyond its current framework. Researchers worldwide are actively exploring theories that might extend or replace the Standard Model, seeking a deeper understanding of the universe's mysteries [3].

In this exploration, the Standard Model not only serves as a comprehensive guide to the known particles and their interactions but also acts as a launching pad for further investigations into the yet-unsolved puzzles that lie at the forefront of modern physics.

1.2 Fundamental Particles and Interactions

The Standard Model of particle physics contains 12 flavors of elementary fermions, plus their corresponding antiparticles, as well as elementary bosons that mediate the forces and the Higgs boson.

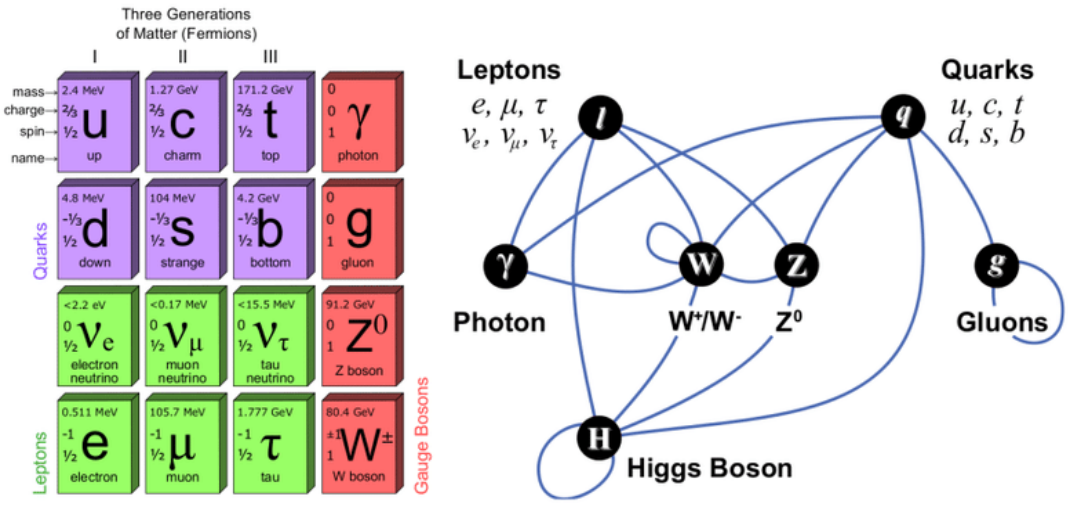


Figure 01: Bosons and Fermions (taked from Wekipedia)

However, the Standard Model is widely considered to be a provisional theory rather than a truly fundamental one, since it is fundamentally incompatible with Einstein's general relativity. There are likely to be hypothetical elementary particles not described by the Standard Model, such as the graviton, the particle that would carry the gravitational force or the sparticles, supersymmetric partners of the ordinary particles.

1.2.1 Elementary particles

Elementary-particle physics studies the fundamental components of matter and their interactions. The standard model, comprising successful mathematical theories of electromagnetic, weak, and strong interactions, likely accurately describes nature down to scales $1/1000th$ the size of an atomic nucleus [4].

Classification of elementary particles

The vast array of particles that exist in the universe can be arranged and studied by classifying particles according to their properties, such as mass, charge, and spin.

In the Standard Model, elementary particles are classified as fermions (spin $s = \frac{1}{2}$) and bosons (spin $s = 1$). Fermions are usually related with matter, whereas bosons are related with fundamental forces (or radiation).

Fermions Fermions obeys the Fermi-Dirac statistics, are named after Enrico Fermi. They are subdivided into quarks and leptons. Quarks are fermions that couple with a gluons to form hadrons (composite particles) such as protons and neutrons. Leptons are those fermions that do not interact with gluons. Electrons are a well-known example of leptons.

- **The first generation** include familiar particles such as electrons, up and down quarks, and their corresponding neutrinos. These particles form the building blocks of ordinary matter and are essential for understanding the properties of atoms and molecules

Generation 1						
Fermion (left-handed)	Symbol	Electric charge	Weak isospin	Weak hypercharge	Color charge	Mass
Electron	e^-	-1	$-1/2$	-1	1	$511keV$
Positron	e^+	$+1$	0	$+2$	1	$511keV$
Electron-neutrino	v_e	0	$+1/2$	-1	1	$< 2eV$
Up quark	u	$+2/3$	$+1/2$	$+1/3$	3	$\tilde{}3MeV$
Up antiquark	$\bar{u}$	$-2/3$	0	$-4/3$	$\bar{3}$	$\tilde{}3MeV$
Down quark	d	$-1/3$	$-1/2$	$+1/3$	3	$\tilde{}6MeV$
Down antiquark	$\bar{d}$	$+1/3$	0	$+2/3$	$\bar{3}$	$\tilde{}6MeV$

Table 01: 1st generation of fermions and their properties [5]

- **The second generation** comprise heavier particles found in the first generation, namely the muon, strange quark, and their respective neutrinos. These particles have the same quantum properties and electric charges as their first-generation counterparts but with significantly greater mass.

Generation 2						
Fermion (left-handed)	Symbol	Electric charge	Weak isospin	Weak hypercharge	Color charge	Mass
Muon	μ^-	-1	$-1/2$	-1	1	$106MeV$
Antimuon	μ^+	$+1$	0	$+2$	1	$106MeV$
Muon-neutrino	v_μ	0	$+1/2$	-1	1	$< 2eV$
Charm quark	c	$+2/3$	$+1/2$	$+1/3$	3	$\tilde{}1.337GeV$
Charm antiquark	$\bar{c}$	$-2/3$	0	$-4/3$	$\bar{3}$	$\tilde{}1.3GeV$
Strange quark	s	$-1/3$	$-1/2$	$+1/3$	3	$\tilde{}100MeV$
Strange antiquark	$\bar{s}$	$+1/3$	0	$+2/3$	$\bar{3}$	$\tilde{}100MeV$

Table 02: 2nd generation of fermions and their properties [5]

- **The third generation** we found the most massive particles yet discovered, namely the tau lepton, bottom quark, and top quark, along with their associated neutrinos. These particles play an essential role in high-energy physics.

Generation 3						
Fermion (left-handed)	Symbol	Electric charge	Weak isospin	Weak hypercharge	Color charge	Mass
Tau lepton	τ^-	-1	$-1/2$	-1	1	$1.78GeV$
Anti-tau lepton	τ^+	$+1$	0	$+2$	1	$1.78GeV$
Tau-neutrino	v_τ	0	$+1/2$	-1	1	$< 2eV$
Top quark	t	$+2/3$	$+1/2$	$+1/3$	3	$171GeV$
Top antiquark	$\bar{t}$	$-2/3$	0	$-4/3$	$\bar{3}$	$171GeV$
Bottom quark	b	$-1/3$	$-1/2$	$+1/3$	3	$\tilde{}4.2GeV$
Bottom antiquark	$\bar{b}$	$+1/3$	0	$+2/3$	$\bar{3}$	$\tilde{}4.2GeV$

Table 03: 3rd generation of fermions and their properties [5]

Bosons Bosons are particles with integer spin, that obey Bose-Einstein statistics and are named after Satyendra Nath Bose and Albert Einstein. In contrast to fermions, which obey Fermi-Dirac statistics, several bosons can occupy the same quantum state. Bosons are often considered as force carrier particles, whereas fermions are usually associated with matter.

Bosons can be elementary, like the photon, or composite, like mesons. Most bosons are composite particles, but in the Standard Model, there are five bosons that are elementary: The gauge bosons (γ, g, $W^\pm$, Z); and the Higgs boson (H) [5].

Gauge boson		Spin	Charge	Mass	Force
photon	γ	1	0	0	electromagnetic
W-boson	$W^{\pm}$	1	$\pm$ 1	80.4 GeV	weak
Z-boson	Z^0	1	0	91.2 GeV	weak
gluons	g	1	0	0	strong
graviton	G	2	0	0	gravitation

Table 04: Bosons of Fundamental Forces(taked from [6])

The photon γ is the mediator of the electromagnetic interaction, while the $W^{\pm}$ and Z^0 bosons are responsible for the weak force. The gluon g mediates the strong interaction, and the Higgs boson H gives mass to other particles.

1.2.2 Fundamental Interactions

There exist four fundamental forces that dictate the interactions between particles and govern the dynamics of the universe: the electromagnetic, weak, and strong, and the gravitational interaction. These forces are described by gauge theories, which provide a mathematical framework for explaining how particles interact with each other.

Gravitational Interaction:

Gravity is an attractive force between all masses. At the classical level, it is fully described in general relativity by the Einstein-Hilbert action

$$\mathcal{L}_{EH} = \int \sqrt{g} R d^4x \tag{1.1}$$

where:

d^4x denotes the volume element of four-dimensional spacetime,

$\sqrt{g}$ is the determinant of the metric tensor g_{uv}, which incorporates the curvature of spacetime,

R represents the Ricci scalar, a scalar quantity that describes the intrinsic curvature of spacetime.

Variation of this action with respect to the metric tensor leads to the Einstein field equations, which describe how spacetime curvature is influenced by the stress-energy tensor T_{uv}

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1.2}$$

Currently, there is no complete quantum theory of gravity available. However, this isn't a big issue in many cases since gravitational interaction is much weaker than other forces, making it negligible in most particle physics phenomenons. Gravity is not included in the Standard Model.

Electromagnetic Interaction:

Quantum Electrodynamics (QED) describes the electromagnetic interaction, which is responsible for interactions between charged particles. Photons, the force carriers of electromagnetism, mediate these interactions. QED is a gauge theory based on the principles of quantum mechanics and special relativity.

The electromagnetic interactions is described in terms of equations of motion, derived from a Lagrangian density consists of a term that describes the free-photon field and an interaction term

describing the force between this photon and a fermionic current,

$$\mathcal{L}_{EM} = i\bar{\Psi}\gamma^{\alpha}\left(\partial_{\alpha} - ieA_{\alpha}\right)\Psi - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1.3}$$

perturbation theory is used to calculate quantities such as scattering amplitudes and cross-sections by expanding them in a series of interaction terms. These calculations are typically done as a series expansion in terms of the coupling constant $e^2/4\pi\hbar c = \alpha \simeq 1/137$ [4], which represents the strength of the interaction between particles.

The Lagrangian density consists of a term that describes the free-photon field,

$$\mathcal{L}_{\gamma} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) \tag{1.4}$$

where

$$F_{\mu\nu}(x) = \frac{\partial A_{\nu}(x)}{\partial x^{\mu}} - \frac{\partial A_{\mu}(x)}{\partial x^{\nu}} \tag{1.5}$$

is the electromagnetic field tensor. $\mathcal{L}_{\gamma}$ is just $\frac{1}{2}[E^2(x) - B^2(x)]$ in more common notation. It is written in terms of the vector potential $A_{\mu}(x)$ because the terms that involve the lepton and its interaction with the electromagnetic field are simplest when written in terms of $A_{\mu}(x)$:

$$\mathcal{L}_{l} = i\bar{\Psi}(x)\gamma^{\alpha}(\frac{\partial}{\partial x^{\alpha}} - ieA_{\alpha}(x))\Psi(x) - m\bar{\Psi}(x)\Psi(x) \tag{1.6}$$

Here, $\Psi(x)$ is a four-component spinor representing the charged fermion, and $\bar{\Psi}$ is the conjugate field defined by

$$\bar{\Psi}(x) = \Psi(x)^{\dagger}\gamma^{0} \tag{1.7}$$

The γ^{α} with $\alpha = (0, 1, 2, 3)$ are the Dirac matrices [4×4 matrices that satisfy the conditions $(\gamma^1)^2 = (\gamma^2)^2 = (\gamma^3)^2 = -(\gamma^0)^2 = -1$ and $\gamma^{\alpha}\gamma^{\beta} = -\gamma^{\beta}\gamma^{\alpha}$ for $\alpha \neq \beta$], m has the dimensions of a mass in the natural units in which $\hbar = c = 1$. If e were zero, the Lagrangian would describe a free lepton; while with $e \neq 0$ the interaction term has the form

$$-eA^{\alpha}(x)j_{\alpha}(x) \tag{1.8}$$

where the current $j_{\alpha}(x)$ is given by

$$j_{\alpha}(x) = -\bar{\Psi}(x)\gamma_{\alpha}\Psi(x) \tag{1.9}$$

The equations of motion show that the current is conserved,

$$\frac{\partial}{\partial x_{\alpha}}j_{\alpha}(x) = 0 \tag{1.10}$$

so that the charge

$$Q = \int d^3r j_0(r, t) \tag{1.11}$$

is a constant of the motion.

The form of the interaction is obtained by making the minimal substitution

$$\frac{\partial}{\partial x^{\alpha}} \rightarrow \nabla_{\alpha} = \frac{\partial}{\partial x^{\alpha}} - eA^{\alpha}(x) \tag{1.12}$$

in the Lagrangian for a free lepton. This minimal coupling follows from a deep principle, local gauge invariance. The requirement that $\Psi(x)$ can have its phase changed locally without affecting the physics of the lepton, that is, invariant under the gauge transformation

$$\Psi(x) \rightarrow e^{-\theta(x)}\Psi(x) \tag{1.13}$$

which, can only be implemented through the introduction of a vector field $A_\alpha(x)$, coupled as in (1.12), and transforming according to

$$A_\alpha(x) \rightarrow A_\alpha(x) - \frac{1}{e}\frac{\partial\theta(x)}{\partial x^\alpha} \tag{1.14}$$

Weak Interactions

The weak force is responsible for processes such as beta decay and neutrino interactions. It is characterized by its short range and its role in changing the flavor of particles through charged and neutral current interactions.

In contrast to the electromagnetic interaction whose form was already contained in classical electrodynamics (CED), it took many decades of experimental and theoretical work to arrive at the next compact phenomenological Lagrangian density describing the weak interactions

$$\mathcal{L}_w^I = \frac{g}{\sqrt{2}}\bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu W_\mu^+ \tag{1.15}$$

Where we can distinguish the following differences:

- The term $\left(1-\gamma^5\right)$ was included in order to formulate a vector-axial current

  $$J^\mu = \bar{\psi}_e\gamma^\mu\left(1-\gamma^5\right)\psi_\nu \tag{1.16}$$

  for describing the parity violation in weak interaction processes.

- Unlike the QED process, which preserves the flavor charge, the weak interaction current changes the flavor (electron to its associated neutrino as an example).

- The photon field was replaced by the $\left(W^\pm, Z^0\right)$ boson, and the coupling constant also changed.

- The same comments hold also for the neutral current

  The interaction terms in the lagrangian are given by

$$\mathcal{L}_w = \frac{g}{\sqrt{2}}\left[\bar{\psi}_e\gamma^\mu(1-\gamma^5)\psi_\nu W_\mu^+ + \bar{\psi}_\nu\gamma^\mu(1-\gamma^5)\psi_e W_\mu^-\right] + \frac{g}{2\cos\theta_W}\left(\bar{\psi}_e\gamma^\mu(g_v - g_a\gamma^5)\psi_e Z_\mu\right) + h.c. \tag{1.17}$$

Where, the charged current interactions are mediated by the $W^\pm$ bosons, and the force changes the flavor of particles involved, like in beta decay, where a neutron decays into a proton, an electron, and an electron antineutrino

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{1.18}$$

It is described by

$$\mathcal{L}_w^{CC} = \frac{g}{\sqrt{2}}\left[\bar{\psi}_e\gamma^\mu(1-\gamma^5)\psi_\nu W_\mu^+ + \bar{\psi}_\nu\gamma^\mu(1-\gamma^5)\psi_e W_\mu^-\right] + h.c. \tag{1.19}$$

while, the neutral current interactions involve the exchange of Z bosons, where the flavor of particles remains unchanged.

$$\mathcal{L}_w^{NC} = \frac{g}{2\cos\theta_W}\left(\bar{\psi}_e\gamma^\mu(g_v - g_a\gamma^5)\psi_e Z_\mu\right) + h.c. \tag{1.20}$$

Historically, the Weak force was firstly described by a poit like interaction

$$\mathcal{L}_w = -\frac{G}{\sqrt{2}}J_\alpha^\dagger(x)J^\alpha(x) \tag{1.21}$$

This form involves vectorial quantities, as originally proposed by E. Fermi [7]. The current $J^{\alpha}(x)$is known as a charged current since it changes (lowers) the electric charge when it acts on a particle. That is, it describes a transition such as

$$\nu_e \rightarrow e^- \tag{1.22}$$

of one particle into another, or the corresponding creation of an $e^-\bar{\nu}_e$ pair.

Similarly, the current $J^{\dagger}_{\alpha}$ describes a charge-raising transition such as

$$n \rightarrow p \tag{1.23}$$

The lagrangian (1.21) describes a (point-like) four-fermi interaction (without propagator), in contrast to electrodynamics, in which the force is transmitted by the exchange of a photon. An additional class of "neutral-current" terms was discovered in 1973 (see Weak Neutral Currents, Currents in Particle Theory)[8].

Both polar and axial vector terms appear ($\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$is a pseudoscalar matrix), so that in the quadratic form (1.17) there will be vector–axial-vector interference terms, indicating parity nonconservation. The discovery of this phenomenon, following the suggestion of T. D. Lee and C. N. Yang in 1956 that reflection invariance in the weak interactions could not be taken for granted but had to be tested, played an important role in the determination of the phe nomenological Lagrangian (1.17) ([9]). The experiments suggested by Lee and Yang all involved looking for a pseudoscalar observable in a weak interaction experiment, and the first of many experiments (C. S. Wu, E. Ambler, R)[10]

Parity voilation Parity violation refers to the phenomenon where certain physical processes do not exhibit symmetry under spatial inversion, meaning they do not behave the same way if left and right are swapped. Discovered in the 1950s through experiments by Chien-Shiung Wu [11], parity violation was observed in weak interactions, such as beta decay, where electrons are emitted asymmetrically relative to the spin of the nucleus. This discovery was pivotal in developing the electroweak theory and the Standard Model of particle physics, highlighting the unique nature of the weak force and reshaping our understanding of fundamental symmetries in nature.

The parity operator performs spatial inversion through the origin:

$$\Psi'(\vec{x},t) = \hat{p}\Psi(\vec{x},t) = \Psi(-\vec{x},t) \tag{1.24}$$

To preserve the normalisation of the wave-function

$$\langle\Psi|\Psi\rangle = \langle\Psi'|\Psi'\rangle = \langle\Psi|\,\hat{p}^{\dagger}\hat{p}\,|\Psi\rangle \tag{1.25}$$

which leads to

$$\hat{p}^{\dagger}\hat{p} = I \rightarrow \hat{p}\ is\ unitary \tag{1.26}$$

but since

$$\hat{p}\hat{p} = I \rightarrow\ \hat{p} = \hat{p}^{\dagger} \rightarrow \hat{p}\ is\ hermitian \tag{1.27}$$

- If the interaction Hamiltonian commutes with $\hat{p}$, parity is an observable conserved quantity
- If $\Psi(\vec{x},t)$ is an eigenfunction of the parity operator with eigenvalue P

$$\hat{p}\Psi(\vec{x},t) = p\Psi(\vec{x},t) \rightarrow \hat{p}\hat{p}\Psi(\vec{x},t) = p\hat{p}\Psi(\vec{x},t) = p^2\Psi(\vec{x},t) \tag{1.28}$$

since $\hat{p}\hat{p} = I, p^2 = 1 \rightarrow$ Parity has eigenvalues $p = \pm 1$

- QED and QCD are invariant under parity.

Strong Interactions

The strength of the coupling that manifests itself in nuclear forces and in the interaction of pions with nucleons is such that perturbation theory, cannot be applied to any field theory of the strong interactions in which the mesons and baryons are the fundamental fields.

The large number of hadronic states strongly suggests a composite structure that cannot be viewed as a perturbation about noninteracting systems. In fact, it is now generally believed that the strong interactions are described by a gauge theory, quantum chromodynamics (QCD), in which the basic entities are quarks rather than hadrons. Nevertheless, prior and parallel to the development of the quark theory a wealth of experimental information concerning the hadrons and their interactions was accumulated.

Quantum Chromodynamics (QCD) is the theory of strong interactions in the standard model. It explains the strong interaction as an interaction between quarks and gluons. According to this theory, each quark carries a color charge which can be of three types: "blue (b)", "green (g)" or "red (r)". Gluons, intermediaries of the strong interaction, carry both a color and an anti-color (for example, blue-antired, or green-antiblue). The interaction of a gluon with a quark can modify the color of the latter. The spinor of each quark has 3 components in color space, in which gluons act

$$q = \begin{pmatrix} q^r \\ q^g \\ q^b \end{pmatrix} \tag{1.29}$$

Quantum chromodynamics describes the strong interaction as a particular gauge group on the color of quarks, called the "group of gauge"$SU(3)_C$,Where C denotes color.

We can formulate the latter by analogy with quantum electrodynamics, starting with the Lagrangian of the.

$$\mathcal{L}_0 = \sum_n \bar{q}_n (i\gamma^\mu \partial_\mu - m_n) q_n \tag{1.30}$$

where q and $\bar{q}$ are the quark field and its conjugate, and the summation is over the different flavors. For a single quark, we have:

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_n) q \tag{1.31}$$

The gauge transformation is defined by:

$$q(x) \to Uq(x) = e^{i\alpha_\alpha(x) T_\alpha} q(x) \tag{1.32}$$

where the α_a are parameters of the $SU(3)$ group, and the generators of the $SU(3)$ group are the matrices $T_a = \lambda_a/2$, where the λ_a $(a = 1..8)$ are the eight Gell-Mann matrices:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{1.33}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},\ \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.34}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{1.35}$$

The matrices T_a satisfy the Lie algebra

$$[T_\alpha, T_\beta] = i f^{\alpha\beta\gamma} T^\gamma \tag{1.36}$$

where $f^{\alpha\beta\gamma}$ matrices, which are completely antisymmetric under index permutation, are the structure constants of the $SU(3)$ group [4].

Invariance under gauge transformations requires the introduction of a boson G_u^a that transform according to

$$G_u^a \rightarrow G_u^a - \frac{1}{g}\partial_u \alpha_a - f_{abc}\alpha_b G_u^c \tag{1.37}$$

which interacts with quark fields through minimal substitution:

$$\partial_u \rightarrow D_u = [\partial_u + igT_a G_u^a] \tag{1.38}$$

So the invariant Lagrangian will have the form:

$$\begin{aligned} \mathcal{L} &= \bar{q}(i\gamma^u D_u - m)q \\ &= \bar{q}(i\gamma^u \partial_u - m)q - g(\bar{q}\gamma^u T_a q)G_u^a \\ &= \mathcal{L}_0 + \mathcal{L}_{int} \end{aligned} \tag{1.39}$$

Finally, we add the Lagrangian density of the free boson:

$$\mathcal{L} = \bar{q}(i\gamma^u \partial_u - m)q - g(\bar{q}\gamma^u T_a q)G_u^a - \frac{1}{4}G_{uv}^a G_a^{uv} \tag{1.40}$$

where G_{uv}^a is the gluonic field tensor, analogous to the electromagnetic field tensor, and it is given by

$$G_{uv}^a = \partial_u G_v^a - \partial_v G_u^a - g f_{abc} G_u^b G_v^c \tag{1.41}$$

The tensor G_{uv}^a has a new property that we can see in the symbolic form of the total Lagrangian [12]

$$\mathcal{L}_{QCD} = (\bar{q}q) + (GG) + g(\bar{q}qG) + g(G^3) + g^2(G^4) \tag{1.42}$$

The first three terms have analogues in quantum electrodynamics; they describe the propagation of free quarks and gluons, as well as the quark-gluon interaction. The last two terms demonstrate the presence of the coupling of three and four gluons in quantum chromodynamics and reflect the fact that gluons have color charges.



Figure 02: Interactions terms in QCD

The final Lagrangian of quantum chromodynamics will be written as:

$$\begin{aligned} L_{QCD} &= \bar{q}_i(i\gamma^u(D_u)_{ij} - m\delta_{ij})q_j - \frac{1}{4}(\partial^u G_a^v - \partial^v G_a^u)(\partial_u G_v^a - \partial_v G_u^a) \\ &\quad + g_s G_a^u \left(\bar{q}_i \gamma^u \frac{\lambda_{ij}^a}{2} q_j\right) \\ &\quad - \frac{g_s}{2} f^{abc}(\partial^u G_a^v - \partial^v G_a^u) - \frac{g_s^4}{4} f^{abc} f_{ade} G_b^u G_c^v G_u^d G_v^e \end{aligned} \tag{1.43}$$

1.3 Feynman rules:

Feynman diagrams are a graphical representation of a term in the perturbative expansion of a transition amplitude defined by incoming and outgoing lines. They are used for calculations in quantum field theory to study scattering processes.

Particles are depicted by lines, which can be drawn in various ways depending on the type of particle being represented. A point where lines connect is called an interaction vertex. The lines fall into three categories: internal lines (propagator), incoming lines, and outgoing lines, representing the initial and final states of the particles.

We will mention a few laws used.

External Lines:

- For a fermion (antifermion) with a momentum p and a spin s:

$$fermion \left\{ \begin{array}{c} incoming(\ ) : u^s(p) \\ outgoing(\ ) : \bar{u}^s(p) \end{array} \right\}$$

$$Antifermion \left\{ \begin{array}{c} incoming(\ ) : \bar{v}^s(p) \\ outgoing(\ ) : v^s(p) \end{array} \right\}$$

- For a boson with polarization ϵ:

$$boson \left\{ \begin{array}{c} incoming(\ ) : \varepsilon_u \\ outgoing(\ ) : \varepsilon_u^* \end{array} \right\}$$

Propagators:

- Each internal line is replaced by:

$$fermion - anti\ - fermion(\ ) : \frac{i(q + mc)}{q^2 - m^2c^2}$$

$$Higgs(\ ) : \frac{i}{k^2 - m_H^2 + i\varepsilon}$$

- Propagator of $W^{\pm}$

$$\begin{aligned} propagator\ of\ W^{\pm}(\ ) \quad &: \quad iD_{F_{\alpha\beta}}(k, M_\omega) = \frac{-i\left(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{M^2c^2}\right)}{k^2 - M_\omega^2 c^2} \\ &= \frac{-ig_{\alpha\beta}}{(M_\omega c)^2}\left(for k^2 \ll (M_\omega c)^2\right) \end{aligned} \tag{1.44}$$

$$photon(\ ) : \frac{-ig_{uv}}{q^2} \tag{1.45}$$

$$Gluon(\ ) : \frac{-ig_{uv}\delta^{\alpha\beta}}{q^2} \tag{1.46}$$

Vertex:

- Each vertex is replaced by:

$$Fermion - photon(\ \ ) : ie\gamma^{\alpha} \tag{1.47}$$

$$Fermion - W^{\pm}(\ \ ) := \frac{-ig}{2\sqrt{2}}\gamma^{\alpha}(1-\gamma^{5}) \tag{1.48}$$

$$Neutrino - Z(\ \ ) : \frac{-ig}{4\cos\theta_W}\gamma^{\alpha}(1-\gamma^{5}) \tag{1.49}$$

$$Fermion - Z(\ \ ) := \frac{-ig}{4\cos\theta_W}\gamma^{\alpha}(1-4\sin^2\theta_W-\gamma^{5}) = \frac{-ig}{2\cos\theta_W}\gamma^{\alpha}(g_V-g_A\gamma^{5}) \tag{1.50}$$

$$Fermion - Higgs(\ \ ) : \frac{-i}{v}m_l \tag{1.51}$$

$$Quark - gluon(\ \ ) := \frac{-ig_s}{2}\lambda^{\alpha}\gamma^{\beta} \tag{1.52}$$

1.4 The Higgs mechanism

The Higgs mechanism is the mechanism through which elementary particles acquire mass. It involves the particle that is connected with the Higgs field, the Higgs boson, and the Higgs field itself, which permeates the universe.

In simple terms, as particles move through the Higgs field, they interact with it, slowing down and gaining mass. This interaction is what gives particles their mass, allowing them to form the building blocks of matter.

In the Standard Model of particle physics, the Higgs field is a fundamental scalar field that gives elementary particles mass via the Higgs mechanism. It is connected to the particle known as the Higgs boson, which was discovered in 2012 at the Large Hadron Collider (LHC) [2].

The Higgs field (ϕ) is a complex scalar doublet under the $SU(2)_L$ gauge group:

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} \tag{1.53}$$

Where $\phi^{+}$and ϕ^{0} are the charged and neutral components, respectively.

The Lagrangian density describing the dynamics of the Higgs field in the Standard Model is:

$$\mathcal{L}_{Higgs} = (\mathcal{D}_u\Phi)^{\dagger}(\mathcal{D}^u\Phi) - V(\Phi) \tag{1.54}$$

where:

- $\Phi(x)$ is the Higgs field.
- $\mathcal{D}_u$ is the covariant derivative, which accounts for the field's interaction with the electroweak gauge fields $W^{\pm}$ and Z as well as the electromagnetic field A_u
- $V(\Phi)$ is the Higgs potential defined by

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4 \tag{1.55}$$

where $\mu^2 < 0$ and $\lambda > 0$. The potential has a minimum at a nonzero value of Φ.

Masses of the Bosons

The Lagrangian for Gauge Boson-Higgs Interaction is given by:

$$\begin{aligned} L_H &= (D_u\phi)^* (D^u\phi) - V(\phi) \\ &= (D_u\phi)^* (D^u\phi) + \mu^2\phi^*\phi - \lambda (\phi^*\phi)^2 \end{aligned} \tag{1.56}$$

where the covariant derivative:

$$D_u = \left[\partial_u + i\frac{g'}{2}B_u + i\frac{g}{2}\tau_a W_u^a\right] \tag{1.57}$$

which acts on a complex $SU(2)$ field:

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} \tag{1.58}$$

The higgs potential

$$V = \mu^2 |\Phi|^2 + \lambda |\Phi|^4 \tag{1.59}$$

$$= \mu^2 \Phi^\dagger\Phi + \lambda \left(\Phi^\dagger\Phi\right)^2 \tag{1.60}$$

Spontaneous Symmetry Breaking For $\mu^2 < 0$, the Higgs field acquires a nonzero vacuum expectation value (VEV), which spontaneously breaks the symmetry $SU(2)$. If we minimize the potential, we find:

$$\frac{\partial V}{\partial \Phi} = 0 \tag{1.61}$$

$$\rightarrow \frac{\partial V}{\partial \Phi} = \mu^2\Phi^\dagger + 2\lambda\left(\Phi^\dagger\Phi\right)\Phi^\dagger = 0$$

$$\rightarrow \mu^2 + 2\lambda\left(\Phi^\dagger\Phi\right) = 0$$

$$\rightarrow \Phi^\dagger\Phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \tag{1.62}$$

So, the minimum occurs at

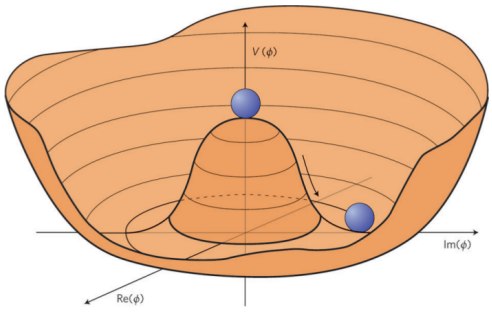


Figure 03: Mexican Hat Potential (from [13])

The vacuum expectation value of the scalar field is

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \upsilon\end{pmatrix} \tag{1.63}$$

where $\upsilon = \sqrt{-\frac{\mu^2}{\lambda}} \approx 246\ GeV$ [14].
then, vacuum containing non-zero fields We can try to fix it by redefinition of our scalar field. We shift it to the minimum at $\pm\upsilon$ by introducing: (if we express $\mathcal{L}$ by replacing $\phi \rightarrow H$, vacume correspond to $H = 0$)

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \upsilon + H\end{pmatrix} \tag{1.64}$$

As we are interested only in the contribution for the gauge boson masses, we omit any Higgs interacting terms in what follows. The piece generating the gauge boson masses is

$$\begin{aligned}
\left(D^u\Phi\right)^\dagger\left(D_u\Phi\right) &= \left|\left(\partial_u + i\frac{g}{2}\tau_a W_u^a + i\frac{g'}{2}B_u\right)\Phi\right|^2 \\
&= \left|\left(\partial_u + i\frac{g}{2}\tau_a W_u^a + i\frac{g'}{2}B_u\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \upsilon\end{pmatrix}\right|^2 \\
&= \frac{\upsilon^2}{4}\left|\left(g\tau_a W_u^a + g'B_u\right)\begin{pmatrix}0\\ 1\end{pmatrix}\right|^2 \\
&= \frac{\upsilon^2}{4}\left|\left(g\left(W_u^1\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + W_u^2\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} + W_u^3\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right) + g'B_u\right)\begin{pmatrix}0\\ 1\end{pmatrix}\right|^2 \\
&= \frac{\upsilon^2}{4}\left|\begin{pmatrix}gW_u^3 + g'B_u & g\left(W_u^1 - iW_u^2\right)\\ g\left(iW_u^2 + W_u\right) & g'B_u - gW_u^3\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}\right|^2 \\
&= \frac{\upsilon^2}{4}\left|\begin{pmatrix}g\left(W_u^1 - iW_u^2\right)\\ \left(g'B_u - gW_u^3\right)\end{pmatrix}\right|^2
\end{aligned} \tag{1.65}$$

$$\begin{aligned}
\left(D^u\Phi\right)^\dagger\left(D_u\Phi\right) &= \frac{\upsilon^2}{4}\left[\left|g\left(W_u^1 - iW_u^2\right)\right|^2 + \left|\left(\left(g'B_u - gW_u^3\right)\right)\right|^2\right] \\
&= \frac{\upsilon^2}{4}\left[g^2\underbrace{\left[\left(W_u^1\right)^2 + \left(W_u^2\right)^2\right]}_{2(W^\pm)^2} + \left(g'B_u - gW_u^3\right)^2\right] \\
&= \underbrace{\frac{\upsilon^2}{4}g^2}_{m_{W^\pm}^2}\left(W^\pm\right)^2 + \frac{\upsilon^2}{8}\left(g'B_u - gW_u^3\right)^2
\end{aligned} \tag{1.66a}$$

The charged vector boson, $W_u^\pm$, and its complex conjugate are defined as $W_u^\pm = \frac{1}{\sqrt{2}}\left(W_u^1 \mp iW_u^2\right)$. Thereby the g^2 term becomes $\frac{\upsilon^2}{4}g^2\left(W^\pm\right)^2 = m_{W^\pm}^2\left(W^\pm\right)^2$. Comparing the first term with the mass term expected for a charged boson, $m_w^2 {W_u}^+ {W_u}^-$, we have

$$m_w = \frac{1}{2}g\upsilon \tag{1.67}$$

The neutral boson mixing term:

$$\frac{\upsilon^2}{4}\left(g'B_u - gW_u^3\right)^2 = \frac{\upsilon^2}{4}\left[g^2\left(W_u^3\right)^2 + g'^2\left(B_u\right)^2 - 2gg'B_uW_u^3\right] \tag{1.68}$$

can be writen in matrix form

$$\frac{v^2}{4}\begin{pmatrix} W_u^3 & B_u \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_u^3 \\ B_u \end{pmatrix} = \frac{v^2}{4}\left[g^2\left(W_u^3\right)^2 - gg'B_uW_u^3 - gg'W_u^3B_u + g'^2\left(B_u\right)^2\right] \tag{1.69}$$

The eigenvalues of this matrix correspond to the masses of the Z boson and the photon A:

$$\left(Z_u\right)^2 = \frac{1}{g^2+g'^2}\left(gW_u^3 - g'B_u\right)^2 = \frac{1}{g^2+g'^2}\left(g^2\left(W_u^3\right)^2 + g'^2\left(B_u\right)^2 - 2gg'W_u^3B_u\right) \tag{1.70}$$

then

$$\begin{aligned} Z_u &= \frac{g}{\sqrt{g^2+g'^2}}W_u^3 - \frac{g'}{\sqrt{g^2+g'^2}}B_u \qquad \longrightarrow m_Z = \frac{v}{2}\sqrt{g^2+g'^2} && \text{(1.71a)} \\ A_u &= \frac{g'}{\sqrt{g^2+g'^2}}W_u^3 + \frac{g}{\sqrt{g^2+g'^2}}B_u \qquad \longrightarrow m_A = 0 && \text{(1.71b)} \\ W_u^{\pm} &= \frac{1}{\sqrt{2}}\left(W_u^1 \mp iW_u^2\right) \qquad \longrightarrow m_W = \frac{gv}{2} && \text{(1.71c)} \end{aligned}$$

Masses of the fermions The same (Higgs) mechanism generates the fermion masses. The Yukawa interaction Lagrangian for a charged lepton l where l can be $(e, , \mu, \tau)$ is given by

$$\mathcal{L}_Y = -y_l\bar{L}_L\phi l_R + h.c. \tag{1.72}$$

where y_l is the Yukawa coupling for the lepton l, $L_L$is the left-handed component of the lepton field, $l_R$is the right-handed component of the lepton field, ϕ is the Higgs field, "$h.c$" stands for the hermitian conjugate.

Substitute the Higgs VEV

$$\begin{aligned} \mathcal{L}_Y &= -y_l\bar{L}_L\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}l_R + h.c. \\ &= -y_l\frac{v}{\sqrt{2}}\bar{L}_Ll_R + h.c. \end{aligned} \tag{1.73}$$

where $\bar{L}_L$ The Dirac adjoint (conjugate transpose and complex conjugate) of the left-handed lepton doublet

$$\bar{L}_L = \left(L_L\right)^{\dagger}\gamma^0 = \begin{pmatrix} v_l \\ l \end{pmatrix}^{+}\gamma^0 = \begin{pmatrix} \bar{v}_l & l \end{pmatrix}_L \tag{1.74}$$

and Spontaneous Symmetry Breaking (SSB) When the Higgs field acquires a non-zero vacuum expectation value (VEV), it breaks the electroweak symmetry. The Higgs doublet's VEV is $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

Combine these terms

$$\bar{L}_L\phi l_R = \begin{pmatrix} \bar{v}_l & l^l \end{pmatrix}_L\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}l_R = \bar{v}_l\phi^+l_R + l_l\phi^0l_R \tag{1.75}$$

Thus, the Yukawa interaction term is

$$L_Y = -y_l\left(\bar{v}_l\phi^+l_R + l_l\phi^0l_R\right) + h.c. \tag{1.76}$$

and replace the Higgs doublet , we find

$$\begin{aligned} L_Y &= -y_l \left(\bar{v}_l \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}^+ l_R + l_l \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} l_R \right) + h.c. \\ &= -y_l \left(\bar{l} \frac{v}{\sqrt{2}} l_R \right) + h.c. \\ &= -y_l \frac{v}{\sqrt{2}} \left(\bar{l}_L l_R + \bar{l}_R l_L \right) \\ &= -y_l \frac{v}{\sqrt{2}} \bar{l} l \end{aligned} \tag{1.77}$$

Since the upper component of the VEV is zero, the term involving $\bar{v}_l \phi^+$ vanishes , and the mass m_l of the charged lepton is:

$$m_l = y_l \frac{v}{\sqrt{2}} \tag{1.78}$$

For each charged lepton Electron (e) Left-handed doublet
Electron Mass Term

$$\begin{aligned} L_Y &= -y_e \left(\bar{L}_{eL} \begin{pmatrix} 0 \\ v \end{pmatrix} e_R \right) \\ &= -y_e \left(\begin{pmatrix} v_e \\ e \end{pmatrix}_L \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e_R \right) \end{aligned} \tag{1.79}$$

where the Left-handed doublet: $L_{eL} = \begin{pmatrix} v_e \\ e \end{pmatrix}_L$ and Right-handed singlet:e_R
Which gives the mass term:

$$m_e = y_e \frac{v}{\sqrt{2}} \tag{1.80}$$

Muon Mass Term:

$$\begin{aligned} L_Y &= -y_\mu \left(\bar{L}_{\mu L} \begin{pmatrix} 0 \\ v \end{pmatrix} \mu_R \right) \\ &= -y_\mu \left(\begin{pmatrix} v_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \mu_R \right) \end{aligned} \tag{1.81}$$

where the Left-handed doublet: $L_{\mu L} = \begin{pmatrix} v_\mu \\ \mu \end{pmatrix}_L$ and Right-handed singlet:μ_R
Which gives the mass term:

$$m_\mu = y_\mu \frac{v}{\sqrt{2}} \tag{1.82}$$

Tau Mass Term:

$$\begin{aligned} L_Y &= -y_\tau \left(\bar{L}_{\tau L} \begin{pmatrix} 0 \\ v \end{pmatrix} \tau_R \right) \\ &= -y_\tau \left(\begin{pmatrix} v_\tau \\ \tau \end{pmatrix}_L \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tau_R \right) \end{aligned} \tag{1.83}$$

where the Left-handed doublet: $L_{\tau L} = \begin{pmatrix} v_\tau \\ \tau \end{pmatrix}_L$ and Right-handed singlet:τ_R

Which gives the mass term:

$$m_\tau = y_\tau \frac{v}{\sqrt{2}} \tag{1.84}$$

The absence of Yukawa Term for Neutrinos within the Standard Model does not allow neutrinos to acquire mass because it lacks right-handed neutrino fields which are essential for forming Yukawa interaction terms analogous to those for charged leptons and quarks.

$$- y_v \frac{v}{\sqrt{2}} \left(\bar{v}_L v_R + \bar{v}_R v_L\right) \tag{1.85}$$

So,

$$m_\nu = 0 \tag{1.86}$$

We will see in the final chapter that this value is inconsistent with the phenomena of neutrino oscillations.

Chapter 2

Cross section, Decay rate and Radiative corrections

2.1 Introduction

The cross sections quantify the Probability that particular particle interactions will take place during collisions. They are essential to comprehending particle experiment results. While, radiative corrections must be taken into account in quantum field theory. These adjustments take into consideration extraneous phenomena that can alter the cross sections, like emission or absorption of virtual photons. Radiative corrections become important when dealing with high energy or in precision experiments, since they help enhance theoretical predictions and experimental results' correctness.

2.1.1 From Field Theory to Observables: Bridging Theory and Experiment

In quantum field theory, the dynamics of particles (fields) are described by the Lagrangian density $\mathcal{L}$. For a theory with both free and interacting fields, the Lagrangian can be split into free and interacting parts:

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} \tag{2.1}$$

From which, we can obtain The Hamiltonian density through a Legendre transformation:

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} \tag{2.2}$$

where $\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$ is the canonical conjugate momentum of the field ϕ. The total Hamiltonian is

$$H = \int d^4x \mathcal{H} \tag{2.3}$$

In the interaction picture, the Hamiltonian can be divided into the free Hamiltonian H_0 and the interaction Hamiltonian H_I:

$$H = H_0 + H_I \tag{2.4}$$

The free Hamiltonian H_0 governs the evolution of the free particles, while the interaction Hamiltonian H_I describes the interactions.

The time evolution operator $U(t, t_0)$ in this picture is given by:

$$U(t, t_0) = T \exp\left(-i \int_{t_0}^{t} dt'\, H_I(t')\right) \tag{2.5}$$

where T is the time-ordering operator, ensuring that times are ordered from left to right in decreasing order.

The Dyson series is the perturbative expansion of the time evolution operator

$$U(t,t_0) = T\exp\left(-i\int_{t_0}^{t} dt'\, H_I(t')\right) \tag{2.6}$$

Expanding this exponential, we obtain:

$$U(t,t_0) = 1 + \sum_{n=1}^{\infty}\frac{(-i)^n}{n!}T\left(\int_{t_0}^{t} dt_1\, H_I(t_1)\int_{t_0}^{t} dt_2\cdots\int_{t_0}^{t} dt_n\, H_I(t_n)\right) \tag{2.7}$$

Simplifying, we get [15]:

$$U(t,t_0) = 1 + \sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1\int_{t_0}^{t} dt_2\cdots\int_{t_0}^{t} dt_n\, T\left(H_I(t_1)H_I(t_2)\cdots H_I(t_n)\right) \tag{2.8}$$

$$U(t,t_0) = T\left(1 - i\textstyle\int_{-\infty}^{+\infty} H_{int}(t)dt + \frac{(-i)^2}{2!}\left(\int_{-\infty}^{+\infty} H_{int}(t)dt\right)^2 + \frac{(-i)^3}{3!}\left(\int_{-\infty}^{+\infty} H_{int}(t)dt\right)^3 + ...\right) \tag{2.9}$$

From which, we can be express the $S-matrix$ or the evolution operator.

$$S = \lim_{\substack{t\to\infty\\ t_0\to-\infty}} U(t,t_0) = I - i\int_{-\infty}^{\infty} H_I(x_I)d^4x_I - \frac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\{H_I(x_1)H_I(x_2)\}d^4x_1d^4x_2 + ... \tag{2.10}$$

This expansion allows us to compute scattering amplitudes and other quantities perturbatively, term by term.

In the case of QED, we can rewrite the dyson expansion as fellow [15]

$$S = \underbrace{I}_{S^{(0)}} + \underbrace{ie\int_{-\infty}^{\infty}(\bar{\psi}A\psi)_{x_1}d^4x_1}_{S^{(1)}} - \underbrace{\frac{1}{2!}e^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\{(\bar{\psi}A\psi)_{x_1}(\bar{\psi}A\psi)_{x_2}\}d^4x_1d^4x_2}_{S^{(2)}} + ... = \sum_{n=0}^{\infty} S^{(n)}. \tag{2.11}$$

To find the transition amplitude $\mathcal{M}_{fi}$, we insert the initial and final states into the Dyson series:

$$\langle f|S|i\rangle = \delta_{fi} + i(2\pi)^4\delta^4(p_f - p_i)\mathcal{M}_{fi} \tag{2.12}$$

In particle physics, the main two observales calculated are the cross section and decay rate:

The cross section is a measure of the probability of a specific interaction occurring when particles meet. It represents the effective area presented by a target particle for a specific interaction.

The differential cross section $d\sigma$ for a 2-to-2 scattering process $a + b \to 1 + 2$ is given by [16]:

$$d\sigma = \frac{|\mathcal{M}_{fi}|^2}{4\sqrt{(p_a\cdot p_b)^2 - m_a^2m_b^2}}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_a + p_b - p_1 - p_2) \tag{2.13}$$

While, The decay rate Γ for a particle decaying from an initial state i to final states f is given by Fermi's Golden Rule [16]:

$$\Gamma = \frac{|\mathcal{M}_{fi}|^2}{2M}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_i - p_1 - p_2) \tag{2.14}$$

2.2 Radiative Corrections:

Radiative corrections are higher-order corrections to particle interactions arising from quantum field theory. They are derived from the Dyson series expansion of the S-matrix (2.11).

Phenomenologically, these corrections result from the complicated interplay between particles and virtual photons in quantum field theory, which causes adjustments in particle attributes such as mass, charge, and interaction strengths.

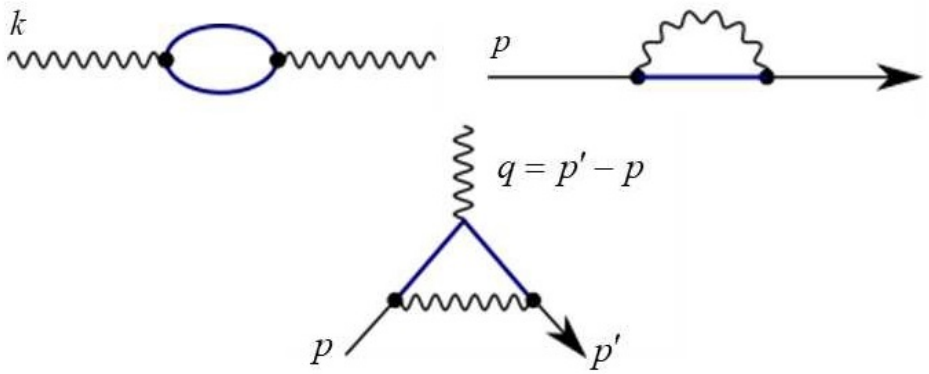


Figure 04: Radiative Corrections in QED

They are critical for understanding and forecasting a wide range of phenomena, including the fine structure of atomic spectra and the anomalous magnetic moments of fundamental particles. Renormalization techniques allow to address the infinities that arise in these corrections, guaranteeing that theoretical predictions match experimental findings with amazing precision.

Radiative corrections result from interactions between charged particles and the quantized electromagnetic field. They include:

- Self-energy corrections occur when charged particles interact with their own electromagnetic fields, causing mass and charge to change. These adjustments are generally divergent, necessitating renormalization.
- Vertex corrections: These occur when the interaction vertices are modified owing to the presence of virtual particles.
- Vacuum polarization occurs when virtual particle-antiparticle pairs exist in the vacuum, affecting the electromagnetic field. This results in changes to the electron's charge and electromagnetic potential.
- Self-interaction corrections: These corrections occur when a particle interacts with its own electromagnetic field, causing corrections to its trajectory.

2.2.1 One-Loop Corrections:

Consider the Lagrangian density for a scalar particle of mass m with cubic self-interaction with coupling constant λ

$$L = \frac{1}{2}(\partial_u \phi)^2 - m^2\phi^2 - \frac{\lambda}{3!}\phi^3 \tag{2.15}$$

We wish to calculate the scattering amplitude for two particles of momenta, p_1 and $p_2$into two particles with momenta p_3 and p_4.

The Feynman rules for the n^{th} order perturbative contribution are:

Fermion Self-energy:

The fermion self-energy refers to the correction that interactions with other particles make to a fermion's propagator. The correction resulting from the interaction of the fermion with the quantum fields can be represented using Feynman diagrams.

Fermion self-energy is a quantum field theory concept that describes the modification of a fermion's propagator due to its interactions with other particles. It represents the corrections to the fermion's mass and wavefunction arising from these interactions.

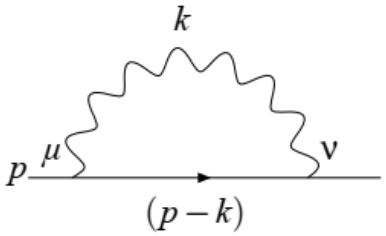


Figure 05: Fermion Self-energy (taken from [18])

Using the rules of QED we have and we change $D = 4 \rightarrow D = 4 - 2\epsilon$ (we reduce the dimension of our spacetime in order to simplify the integrals),

$$\Sigma(p^2, m) = i\mu^{(4-d)} \int \frac{d^d k}{(2\pi)^d} (-ie\gamma^\mu) \frac{i(\gamma.(p-k)+m)}{(p-k)^2 - m^2} (-ie\gamma^v) \frac{ig_{\mu v}}{k^2} \tag{2.16}$$

When we introduce Feynman parameters, we obtain [18]

$$\Sigma(p^2, m) = -ie^2\mu^{(4-d)} \int \frac{d^d k}{(2\pi)^d} \int_0^1 d\alpha \int_0^1 d\beta \delta(1-\alpha-\beta) \frac{\gamma^\mu(\gamma.(p-k)+m)\gamma_\mu}{(k^2(\alpha+\beta) - 2p.k\alpha + (p^2-m^2)\alpha)^2} \tag{2.17}$$

Now shift $k \rightarrow k + p\alpha$

$$\Sigma(p^2, m) = -ie^2\mu^{(4-d)} \int \frac{d^d k}{(2\pi)^d} \int_0^1 d\alpha \frac{\gamma^\mu(\gamma.p(1-\alpha)+m)\gamma_\mu}{(k^2 + p^2(\alpha(1-\alpha) - m^2\alpha)^2} \tag{2.18}$$

We have neglected a term

$$-ie^2\mu^{(4-d)} \int \frac{d^d k}{(2\pi)^d} \int_0^1 d\alpha \frac{\gamma^\mu \gamma.k \gamma_\mu}{(k^2 + p^2(\alpha(1-\alpha) - m^2\alpha)^2} \tag{2.19}$$

setting $d = 4 - 2\varepsilon$, using $\gamma^\mu\gamma.p\gamma_\mu = -2(1-\varepsilon)\gamma.p\,, \gamma^\mu\gamma_\mu = 4 - 2\varepsilon$ *and the result from we have*

$$\Sigma(p^2, m) = -\frac{e^2}{(16\pi^2)} \Gamma(\varepsilon) \int_0^1 d\alpha [2(1-\varepsilon)(1-\alpha)\gamma.p - (4-2\varepsilon)\, m] \left(\frac{4\pi\mu^2}{m^2\alpha - p^2\alpha(1-\alpha)} \right)^\varepsilon \tag{2.20}$$

Finally, by executing the final step, we can return to our initial dimension.

$$\begin{aligned} \Sigma(p^2, m) &= -\frac{e^2}{(16\pi^2)} \left[\int_0^1 d\alpha (2(1-\alpha)\gamma.p - 4m) \right] \left(\frac{1}{\varepsilon} + \ln(4\pi) - \gamma_E \right) \\ &\quad + \int_0^1 d\alpha \left(2\gamma.p(1-\alpha) - 2m + (2(1-\alpha)\gamma.p - 4m) \ln \left(\frac{m^2\alpha - p^2\alpha(1-\alpha)}{\mu^2} \right) \right) \end{aligned} \tag{2.21}$$

and for the wavefunction renormalization constant [18]

$$\begin{aligned} Z_2 &= 1 + \frac{\alpha}{4\pi} \left[1 - \frac{1}{\varepsilon} - \ln(4\pi) + \gamma_E + 2\int_0^1 (1-\alpha) \ln \left(\frac{m^2\alpha^2}{\mu^2} \right) + 4\int_0^1 d\alpha \frac{\alpha(1-\alpha^2)}{\alpha^2 + (1-\alpha)\lambda^2/m^2} \right] \qquad (2.22) \\ &= 1 + \frac{\alpha}{4\pi} \left[1 - \frac{1}{\varepsilon} - \ln \left(\frac{\mu^2}{m^2} \right) + \gamma_E - \ln(4\pi) - 4 + 2\ln\left(m^2/\lambda^2\right) \right] \qquad (2.23) \end{aligned}$$

Photon Self-energy (Vacuum polarization) :

The fundamental idea in quantum field theory explains how interaction between virtual particle-antiparticle pairs, such electron-positron pairs, modifies the photon propagator. This phenomenon occurs because the existence of an electromagnetic field in the vacuum can produce temporary pairs of particles and antiparticles that alter photon propagation.

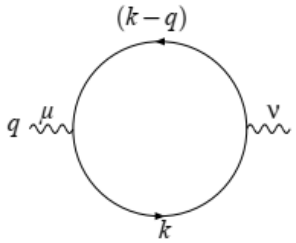


Figure 06: Photon Self-energy (taken from [18])

The photon self-energy $\Pi^{\mu\nu}(q^2)$ is, in general, a two-rank tensor, which is formed from the four momentum of the photon ,q^μ and the (invariant) metric tensor. It must therefore have the form [18]

$$\Pi^{\mu\nu}(q^2) = A(q^2)g^{\mu\nu} + B(q^2)q^\mu q^\nu. \tag{2.24}$$

On the other hand $\Pi^{\mu\nu}(q^2)$ obeys a Ward identity [18]

$$q_\mu \Pi^{\mu\nu}(q^2) = 0 \tag{2.25}$$

This can be seen by writing

$$\frac{1}{(\gamma.k - m)} q^\mu \gamma_\mu \frac{1}{(\gamma.(k-q) - m)} - \frac{1}{(\gamma.k - m)} \tag{2.26}$$

The renormalized propagator is [18]

$$-i\frac{Z_3\left[\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - (\xi_R - 1)\frac{q^\mu q^\nu}{q^2}\right]}{q^2},$$

Where

$$(\xi_R - 1) = \frac{(\xi - 1)}{Z_3}.$$

Inserting the Feynman rules, we get

$$\Pi^{\mu\nu}(q^2) = -i\mu^{4-d}\int \frac{d^dk}{(2\pi)^d} Tr[(-ie\gamma^\mu)\frac{i(\gamma.(k-q)+m)}{((k-q)^2 - m^2)}(-ie\gamma^\nu)\frac{i(\gamma.k+m)}{k^2 - m^2} \tag{2.27}$$

Using $d = 4 - 2\varepsilon$ we obtain

$$\begin{aligned} -q^2\Pi(q^2) &= -\frac{e^2}{4\pi^2}\int_0^1 d\alpha \left(\frac{4\pi\mu^2}{m^2 - q^2\alpha(1-\alpha)}\right)^\epsilon [\Gamma(\varepsilon - 1)\left(\frac{1}{2}(4-2\varepsilon) - 1\right)\left(q^2\alpha(1-\alpha) - m^2\right) \\ &\quad -\Gamma(\varepsilon)\left(q^2\alpha(1-\alpha) + m^2\right)] \end{aligned} \tag{2.28}$$

Using Feynmann parametrezation, we get

$$\Pi(q^2) = -\frac{e^2}{2\pi^2}\Gamma(\varepsilon)\int_0^1 d\alpha\alpha(1-\alpha)\left(\frac{4\pi\mu^2}{m^2 - q^2\alpha(1-\alpha)}\right)^\epsilon \tag{2.29}$$

Now, we expand the expression of Π in terms of ϵ, keeping only the terms of orders ϵ^{-1} and ϵ^0, because the other terms will vanish as $\epsilon \to 0$ (when we return to our original dimension), so we get

$$\Pi(q^2) = -\frac{e^2}{12\pi^2}[\frac{1}{\epsilon} + \ln(4\pi) - \gamma_E - 6\int_0^1 d\alpha\alpha(1-\alpha)\left(\frac{4\pi\mu^4 m^2 - q^2\alpha(1-\alpha)}{\mu^2}\right) \tag{2.30}$$

We define Z_3 to be $1+\ \Pi(0)$ so that we have [18]

$$Z_3 = 1 - \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + \ln(4\pi) - \gamma_E - \ln\left(\frac{m^2}{\mu^2}\right)\right] \tag{2.31}$$

and the renormalized photon self energy

$$\Pi_R(q^2) = \Pi(q^2) - (Z_3 - 1), \tag{2.32}$$

The Vertex Function:

The vertex function is a fundamental concept in quantum field theory, particularly in quantum electrodynamics (QED). It describes the interaction between a fermion and a photon at a vertex, taking into account the effects of quantum corrections .

The fermions have momenta p and p' and the photon has momentum $q = p' - p$. In general, we will have processes (such as Compton scattering of photons off electrons) in which one of the fermion legs are internal and therefore off-shell. Here we restrict ourselves to fermion scattering in which both the fermion legs are on-shell,$i.e$ $.p^2 = p'^2 = m^2$.

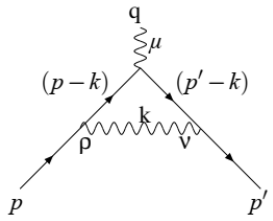


Figure 07: Vertex Correction (from [18])

Using the Feynman rules the vertex correction factor $\Gamma^\mu(p,p')$ is given by

$$\Gamma^\mu(p,p') = \mu^{4-d}\int\frac{d^dk}{(2\pi)^d}(-ie\gamma^\nu)\frac{i(\gamma.(p'-k)+m)}{((p'-k)^2-m^2)}(\gamma^\mu)\frac{i(\gamma.(p-k)+m)}{((p-k)^2-m^2)}(-ie\gamma^\rho)\frac{-ig_{\mu\nu}}{(k^2-\lambda^2)} \tag{2.33}$$

Under Feynmann parametrezation, we get [18]

$$\Gamma^\mu(p,p') = -i2e^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\int_0^1 d\alpha d\beta d\gamma\frac{\delta(1-\alpha-\beta-\gamma)\mathcal{N}}{(k^2-2k.(p\alpha+p'\beta)-\lambda^2\gamma)^3}, \tag{2.34}$$

where $\mathcal{N}$ is the numerator

$$\mathcal{N} = \gamma^\nu(\gamma.(p'-k)+m)\gamma^\mu(\gamma.(p-k)+m)\gamma_\nu$$

transfer $k \to k + p\alpha + p'\beta$ to acquire

$$\Gamma^\mu(p,p') = -i2e^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\int_0^1 d\alpha d\beta d\gamma\frac{0(1-\alpha-\beta)\mathcal{N}_2+\mathcal{N}_0}{(k^2-\lambda^2(1-\alpha-\beta)-m^2(\alpha+\beta)^2+q^2\alpha\beta)^3}, \tag{2.35}$$

The $\mathcal{N}_0$ term does not lead to an ultraviolet divergence and may be calculated in four dimensions †. We have

$$\mathcal{N}_0 = \gamma^\nu(\gamma.p'(1-\beta)) - \gamma.p\alpha + m)\gamma^\mu(\gamma.p(1-\alpha)) - \gamma.p'\alpha + m)\gamma_\nu \tag{2.36}$$

or ,we may write as

$$\mathcal{N}_0 = -2(\gamma.p(1-\alpha) - \gamma.p'\beta)\gamma^\mu(\gamma.p'(1-\beta) - \gamma.p\alpha) - 2m^2\gamma^\mu + 4m(1-\alpha-\beta)(p+p')^\mu \tag{2.37}$$

where we have used the symmetry under $\alpha \to \beta$.
Using the term $q^2\gamma^u$ the numerator $\mathcal{N}_0$ becomes

$$\begin{aligned} \mathcal{N}_0 &= (8(1-\alpha-\beta) - 2 - 2(1-\alpha-\beta)^2)m^2 - 2(1-\alpha)(1-\beta)q^2)\gamma^u \\ &\quad +4im((1-\alpha-\beta) - (1-\alpha)(1-\beta))q_v\sigma_{uv} \end{aligned} \tag{2.38}$$

The counterterm associate wih the vertex is $(Z_1-1)\gamma^u$, so we write

$$\Gamma^u_R = \Gamma^u + (Z_1-1)\gamma^u \tag{2.39}$$

Where Γ^u_R is the finite renormalized vertex correction.
The coupling at zero momentum transfer is the standard definition of the renormalized electromagnetic coupling. Stated otherwise, we need to select the renormalization constant Z_1 such that $\Gamma^u_R(p, p=0$, in order to obtain for [18]

$$Z_1 = 1 + \frac{e^2}{16\pi^2}\left[\begin{array}{c} -\frac{1}{\varepsilon} - \ln(4\pi) + \gamma_E - \ln\left(\frac{\mu^2}{m^2}\right) + 2 \\ +4\int d\alpha d\beta\theta(1-\alpha-\beta)\left\{\ln(\alpha+\beta) + m^2\frac{2(1-\alpha-\beta)-(1-\alpha-\beta)^2}{m^2(\alpha+\beta)^2+\lambda^2(1-\alpha-\beta)^2}\right\} \end{array}\right] \tag{2.40}$$

The last integral has an infrared divergence as $\lambda \to 0$
The integral over α and β is most easily performed by the change of variables $\alpha = \rho\omega$, $\beta = \rho(1-\omega)$ We now have

$$\begin{aligned} Z_1 &= 1 + \frac{\alpha}{4\pi}\left[-\frac{1}{\varepsilon} - \ln(4\pi) + \gamma_E - \ln\left(\frac{\mu^2}{m^2}\right) + 2 + 4\int_0^1 \rho d\rho\left\{\ln(\rho) + \frac{2(1-\rho) - 1 - (1-\rho)^2}{\rho^2 + (1-\rho)\lambda^2/m^2}\right\}\right] \\ &= 1 + \frac{\alpha}{4\pi}\left[-\frac{1}{\varepsilon} - \ln(4\pi) + \gamma_E - \ln\left(\frac{\mu^2}{m^2}\right) - 4 + 2\ln\left(\frac{m^2}{\lambda^2}\right)\right] \end{aligned} \tag{2.41}$$

Examination of 2.22shows that we have

$$Z_1 = Z_2.$$

The renormalized vertex function has a term proprtional to γ^μ and a term proprtional to $q_v\sigma^{\mu v}$, and may be written as

$$\Gamma^\mu_R = \gamma^\mu F_1(q^2) + \frac{i}{2m}q_v\sigma^{\mu v}F_2(q^2) \tag{2.42}$$

The functions F1 and F2 which depend on q2 for the case of on-shell fermion legs are known as the"electric" and "magnetic" form factors respectively.
For $q^2 >> m^2$ we have

$$F_1(q^2) \to 1 - \frac{\alpha}{2\pi}\left[\left(\ln\left(\frac{-q^2}{m^2}\right) - 1\right)\ln\left(\frac{m^2}{\lambda^2}\right) + \ln\left(\frac{-q^2}{m^2}\right) - 2\right] \tag{2.43}$$

$$F_2 \to \frac{q^2}{m^2} \tag{2.44}$$

For $q^2 << m^2$ we have

$$F_1(q^2) \to 1 - \frac{\alpha}{3\pi}\frac{q^2}{m^2}\left[\frac{1}{2}\ln\left(\frac{m^2}{\lambda^2}\right) - \frac{3}{8}\right] \tag{2.45}$$

$$F_2(q^2) \rightarrow \frac{\alpha}{2\pi} \tag{2.46}$$

Using the Feynman rules the vertex correction factor, $\Gamma^\mu(p,p')$ is given (in $d-$dimensions and in Feynman gauge) by

$$\Gamma^\mu(p,p') = -ie^2\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\int_0^1 d\alpha d\beta d\gamma\frac{\delta(1-\alpha-\beta-\gamma)N}{(k^2-2k.(p\alpha+p'\beta)-\lambda^2\gamma)^3}, \tag{2.47}$$

where N is the numerator

$$N = \gamma^\nu(\gamma.(p'-k)+m)\gamma^\mu(\gamma.(p-k)+m)\gamma_\nu$$

The counterterm associate wih the vertex is $(Z_1-1)\gamma^\mu$, so we write

$$\Gamma^\mu_R = \Gamma^\mu + (Z_1-1)\gamma^\mu, \tag{2.48}$$

where Γ^μ_R is the finite renormalized vertex correction.

Chapter 3

Neutrino Phenomenology

This chapter focuses on two key aspects of neutrino phenomenology: oscillations and the seesaw mechanism. Neutrino oscillations reveal their possibility to change flavor. Meanwhile, the seesaw mechanism offers an alternative approach to understanding neutrino masses and their implications for oscillatory behavior. Together, these phenomena provide crucial insights into our understanding of neutrino behavior and their fundamental role in the universe.

History and discovery of Neutrino

Neutrinos were first hypothesized by Pauli in 1930 [19], to explain the apparent violation of energy conservation in beta decay.

$$n \rightarrow p + e^{-} + \bar{\nu}_e \tag{3.1}$$

Then, in 1956, C. Cowan and F. Reines detected neutrinos emitted from a nuclear reactor, confirming their existence experimentally [20].

3.1 Solar neutrino problem

The Sun is a natural nuclear fusion reactor, powered by a proton–proton chain reaction which converts four hydrogen nuclei (protons) into alpha particles, neutrinos, positrons and energy as illustrated in next figure.

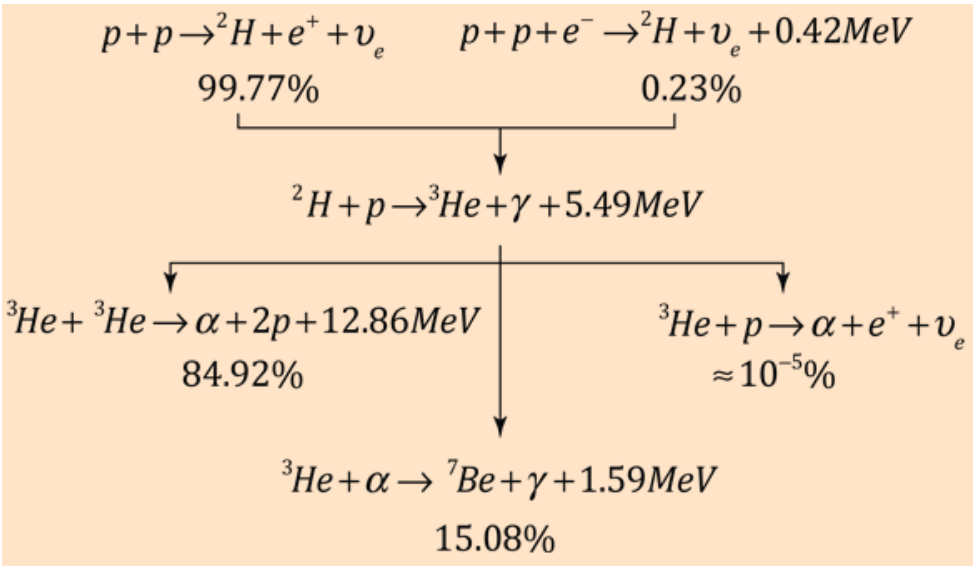


Figure 08: PP Chain (taken from [21])

The excess energy is released as γ rays and as kinetic energy of the particles and as neutrinos ,which travel from the Sun's core to Earth.

In 1968, Ray Davis and colleagues [22], conducted pioneering experiments to measure solar neutrinos. Their work utilized a large tank of chlorine located deep underground in the Homestake mine in South Dakota, aimed at minimizing background interference from cosmic rays. Chlorine atoms in the tank could capture neutrinos, undergoing a reaction that converted them into argon.

$$\begin{aligned} \nu_e + Cl^{37} &\rightarrow Ar^{37} + e^- && (3.2)\\ &\Longleftrightarrow \nu_e + n \rightarrow p + e^- && (3.3) \end{aligned}$$

The Davis experiment, for which he was awarded the Nobel Prize in 2002, collected argon atoms over several months. However, the total accumulation was only about a third of what Bahcall had predicted [23]. Thus was born the famous **solar neutrino problem** (the number of electron neutrinos arriving from the Sun was between one third and one half the number predicted by the Standard Solar Model).

The solar neutrino problem, a longstanding discrepancy, persisted unresolved for about thirty years until the discovery of neutrino oscillation and mass provided a resolution.

3.1.1 Neutrino Oscillation

In 1968, B. Pontecorvo suggested a simple explanation for the solar neutrino problem [24, 25]. He proposed that the electron neutrinos produced by the sun are converted into a different species (muon neutrinos, say, or even antineutrinos), to which Davis' experiment was insensitive [26]. This is the mechanism we now call neutrino oscillation.

Neutrino Oscillation (Simplified Two-Flavor Case) The theory is basically the quantum mechanics of mixed states, which itself is almost identical to the classical theory of coupled oscillators. Consider the case of just two neutrino types - say, ν_e and ν_μ. If one can spontaneously convert into the other, it means that neither is an eigenfunction of the Hamiltonian. The true stationary states for the system are evidently some orthogonal linear combinations:

$$\begin{aligned} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \\ &\rightarrow \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} && (3.4) \end{aligned}$$

So,

$$\begin{aligned} |\nu_e\rangle &= \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle && (3.5)\\ |\nu_\mu\rangle &= -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle && (3.6) \end{aligned}$$

According to the Schrodinger equation, these eigenstates have the time dependence:

$$\nu_1(t) = \nu_1(0)\, e^{-\frac{i}{\hbar}E_1 t}, \qquad \nu_2(t) = \nu_2(0)\, e^{-\frac{i}{\hbar}E_2 t} \tag{3.7}$$

Assuming a neutrino is produced originally as an electron neutrino at time $t = 0$,

$$|\nu(0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \tag{3.8}$$

its state at a later time t will be:

$$|\nu_e(t)\rangle = \cos\theta e^{-\frac{i}{\hbar}E_1 t}\,|\nu_1\rangle + \sin\theta e^{-\frac{i}{\hbar}E_2 t}\,|\nu_2\rangle \tag{3.9}$$

The probability for the final state to be a muonic neutrino is

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= |\langle \nu_\mu | \nu(t) \rangle|^2 \\ &= \left| [\langle \nu_1 | (-\sin\theta) + \langle \nu_2 | (\cos\theta)] \left[\cos\theta e^{-\frac{i}{\hbar}E_1 t} |\nu_1\rangle + \sin\theta e^{-\frac{i}{\hbar}E_2 t} |\nu_2\rangle \right] \right|^2 \\ &= \sin^2(2\theta) \sin^2\left(\frac{\Delta E t}{2\hbar}\right) \end{aligned} \tag{3.10}$$

Which gives the probability that the initial electron neutrino $|\nu_e\rangle$ has oscillated into a muon neutrino $|\nu_\mu\rangle = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$ after traveling a distance $L = ct$.

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= |\langle \nu_\mu | \nu(t) \rangle|^2 = \left| [\langle \nu_1 | (-\sin\theta) + \langle \nu_2 | (\cos\theta)] \left[\cos\theta e^{-\frac{i}{\hbar}E_1 t} |\nu_1\rangle + \sin\theta e^{-\frac{i}{\hbar}E_2 t} |\nu_2\rangle \right] \right|^2 \\ &= \left| \left(-\sin\theta\cos\theta e^{-\frac{i}{\hbar}E_1 t} + \cos\theta\sin\theta e^{-\frac{i}{\hbar}E_2 t} \right) \right|^2 = (\sin\theta\cos\theta)^2 \left| \left(e^{-\frac{i}{\hbar}E_2 t} - e^{-\frac{i}{\hbar}E_1 t} \right) \right|^2 \\ &= \frac{1}{4}\sin^2(2\theta)\, 4\sin^2\left(\frac{\Delta E t}{2\hbar}\right) \end{aligned} \tag{3.11}$$

Let $(\Delta E = E_2 - E_1)$. Then,

$$\begin{aligned} e^{-(i/\hbar)E_2 t} - e^{-(i/\hbar)E_1 t} &= e^{-(i/\hbar)E_1 t}\left(e^{-(i/\hbar)\Delta E t} - 1 \right) \\ &\to \left| e^{-(i/\hbar\surd)E_2 t} - e^{-(i/\hbar)E_1 t} \right|^2 = \left| e^{-(i/\hbar)E_1 t}\left(e^{-(i/\hbar)\Delta E t} - 1 \right) \right|^2 \end{aligned} \tag{3.12}$$

Since $e^{-(i/\hbar)E_1 t}$ is a phase factor and has a magnitude of 1:

$$\left| e^{-(i/\hbar)E_1 t}\left(e^{-(i/\hbar)\Delta E t} - 1 \right) \right|^2 = \left| e^{-(i/\hbar)\Delta E t} - 1 \right|^2 \tag{3.13}$$

Let $z = e^{-(i/\hbar)\Delta E t}$.Then

$$z = e^{-(i/\hbar)\Delta E t} = \cos\left(\frac{\Delta E t}{\hbar}\right) - i\sin\left(\frac{\Delta E t}{\hbar}\right) \tag{3.14}$$

Thus, we need to compute:

$$|z - 1|^2 = \left| \cos\left(\frac{\Delta E t}{\hbar}\right) - i\sin\left(\frac{\Delta E t}{\hbar}\right) - 1 \right|^2 \tag{3.15}$$

For a complex number $(a + ib)$, the magnitude squared is $(a^2 + b^2)$.

$$\left| \cos\left(\frac{\Delta E t}{\hbar}\right) - 1 - i\sin\left(\frac{\Delta E t}{\hbar}\right) \right|^2 = \left(\cos\left(\frac{\Delta E t}{\hbar}\right) - 1 \right)^2 + \sin^2\left(\frac{\Delta E t}{\hbar}\right) \tag{3.16}$$

Use the trigonometric identity $\cos^2 x + \sin^2 x = 1$:

$$\begin{aligned} \left(\cos\left(\frac{\Delta E t}{\hbar}\right) - 1 \right)^2 + \sin^2\left(\frac{\Delta E t}{\hbar}\right) &= \cos^2\left(\frac{\Delta E t}{\hbar}\right) - 2\cos\left(\frac{\Delta E t}{\hbar}\right) + 1 + \sin^2\left(\frac{\Delta E t}{\hbar}\right) \\ &= \left(\cos^2\left(\frac{\Delta E t}{\hbar}\right) + \sin^2\left(\frac{\Delta E t}{\hbar}\right) \right) - 2\cos\left(\frac{\Delta E t}{\hbar}\right) + 1 \\ &= 1 - 2\cos\left(\frac{\Delta E t}{\hbar}\right) + 1 = 2 - 2\cos\left(\frac{\Delta E t}{\hbar}\right) \\ &= 2\left(1 - \cos\left(\frac{\Delta E t}{\hbar}\right) \right) \end{aligned} \tag{3.17}$$

So

$$\left|e^{-(i/\hbar)E_2 t} - e^{-(i/\hbar)E_1 t}\right|^2 = 2\left(1 - \cos\left(\frac{\Delta E t}{\hbar}\right)\right) \tag{3.18}$$

This can also be written using the trigonometric identity $\left(1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)\right)$:

$$\left|e^{-(i/\hbar)E_2 t} - e^{-(i/\hbar)E_1 t}\right|^2 = 4\sin^2\left(\frac{\Delta E t}{2\hbar}\right) \tag{3.19}$$

on the other hand

$$(\sin\theta\cos\theta)^2 = \frac{1}{4}\sin^2(2\theta) \tag{3.20}$$

You see why they are called neutrino oscillations: ν_e will convert to ν_μ and then back again, sinusoidally, just as coupled oscillators go back and forth between the normal modes.

In this theory the electron and muon neutrinos themselves do not have well-defined energies - or masses; the 'mass eigenstates' are ν_1 and ν_1, with masses m_1 and m_2.

$$E^2 = |\mathbf{p}|^2 c^2 + m^2 c^4 = |\mathbf{p}|^2 c^2 \left(1 + \frac{m^2 c^2}{|\mathbf{p}|^2}\right) \tag{3.21}$$

$$\rightarrow \quad E \approx |\mathbf{p}| c \left(1 + \frac{1}{2}\frac{m^2 c^2}{|\mathbf{p}|^2}\right) = |\mathbf{p}| c + \frac{m^2 c^3}{2|\mathbf{p}|} \tag{3.22}$$

$$\Delta E = E_2 - E_1 \approx \frac{m_2^2 c^3 - m_1^2 c^3}{2|\mathbf{p}|} \approx \frac{\left(m_2^2 - m_1^2\right)}{2E} c^4 \tag{3.23}$$

and hence

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= \left[\sin(2\theta)\sin\left(\frac{\Delta E}{2\hbar}t\right)\right]^2 = \left[\sin(2\theta)\sin\left(\frac{\left(m_2^2 - m_1^2\right)c^4}{4\hbar E}t\right)\right]^2 \\ &= \left[\sin(2\theta)\sin\left(\frac{\left(m_2^2 - m_1^2\right)c^3}{4\hbar E}ct\right)\right]^2 \\ &= \left[\sin(2\theta)\sin\left(\frac{\left(m_2^2 - m_1^2\right)c^3}{4\hbar E}x\right)\right]^2 \end{aligned} \tag{3.24}$$

In particular, after a distance

$$\begin{aligned} \frac{\left(m_2^2 - m_1^2\right)c^3}{4\hbar E} L &= \frac{\pi}{2} \\ L &= \frac{2\pi\hbar E}{\left(m_2^2 - m_1^2\right)c^3} \end{aligned} \tag{3.25}$$

the probability of conversion hits a maximum, $\sin^2(2\theta)$, and at $2L$ they are all back to electron neutrinos.

The neutrino appears as a superposition of two wave packets with different masses.

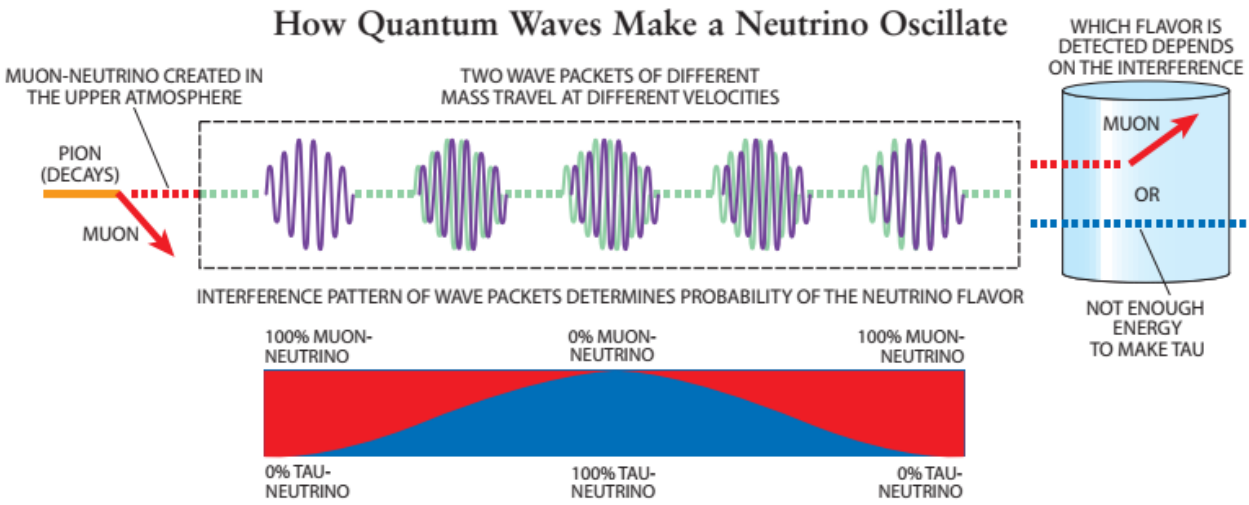


Figure 09: Wave interference and oscillations. (taken from [27])

These wave packets travel at different speeds, causing the lighter one to move ahead of the heavier one. As they propagate, their waves interfere with each other, creating an interference pattern that determines the likelihood of detecting a muon neutrino (red) or a tau neutrino (blue) at any given point along their path (bottom).

Neutrino Oscillation (Three-Flavor Case) The fundamental concept is this: neutrinos interact as flavor eigenstates, meaning they manifest in distinct types (electron, muon, tau). However, during propagation, they behave as eigenstates of the free-particle Hamiltonian, which are mass eigenstates. This dual nature leads to a complex oscillatory evolution where three different masses interact like coupled oscillators, analogous to the beats produced by such systems.

A similar mixing phenomenon occurs with quarks, where the familiar flavors (down, strange, bottom) are the mass eigenstates, and the 'weak eigenstates' (d', s', b') are rotated counterparts analogous to neutrinos. In the quark sector, the CKM matrix describes this relationship between weak and mass eigenstates. For leptons, a corresponding construct known as the 'PMNS matrix' (Pontecorvo–Maki–Nakagawa–Sakata matrix) serves a similar purpose, connecting weak and mass eigenstates and elucidating the behavior of neutrino flavors over distance and time :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{3.26}$$

Here, U is the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix that relates flavor states to mass states. It can be expressed by [16]

$$\begin{aligned} PMNS\ matrix &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{-i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{-i\delta} & c_{13}c_{23} \end{pmatrix} \end{aligned} \tag{3.27}$$

Each mass eigenstate evolves in time according to the Schrodinger equation. For a mass eigenstate

ν_i with energy E_i,

$$\nu_i(t) = \nu_i(0)\, e^{-\frac{i}{\hbar}E_i t} \tag{3.28}$$

An electron neutrino ν_e produced at $t = 0$ is a superposition of mass states:

$$|\nu(0)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle \tag{3.29}$$

As time evolves, each component evolves differently:

$$|\nu_e(t)\rangle = U_{e1}e^{-\frac{i}{\hbar}E_1 t}|\nu_1\rangle + U_{e2}e^{-\frac{i}{\hbar}E_2 t}|\nu_2\rangle + U_{e3}e^{-\frac{i}{\hbar}E_3 t}|\nu_3\rangle \tag{3.30}$$

To find the probability of detecting a different flavor ν_μ at time t, we project $|\nu(t)\rangle$ onto $|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle$:

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= |\langle\nu_\mu|\,|\nu_e(t)\rangle|^2 \\ &= \left|\left[\langle\nu_1|U^*_{\mu 1} + \langle\nu_2|U^*_{\mu 2} + \langle\nu_3|U^*_{\mu 3}\right]\left[U_{e1}e^{-\frac{i}{\hbar}E_1 t}|\nu_1\rangle + U_{e2}e^{-\frac{i}{\hbar}E_2 t}|\nu_2\rangle + U_{e3}e^{-\frac{i}{\hbar}E_3 t}|\nu_3\rangle\right]\right|^2 \\ &= \left|U^*_{\mu 1}U_{e1}e^{-\frac{i}{\hbar}E_1 t} + U^*_{\mu 2}U_{e2}e^{-\frac{i}{\hbar}E_2 t} + U^*_{\mu 3}U_{e3}e^{-\frac{i}{\hbar}E_3 t}\right|^2 \end{aligned} \tag{3.31}$$

Experimental Confirmation Experimental confirmation of neutrino oscillations was a significant achievement in particle physics, indicating that neutrinos have mass and can oscillate between different flavors as they travel through space. Here are some key experiments that have confirmed neutrino oscillations:

- Super-Kamiokande (1998): Observed a deficit in muon neutrinos from atmospheric interactions, suggesting oscillations into tau neutrinos [28].

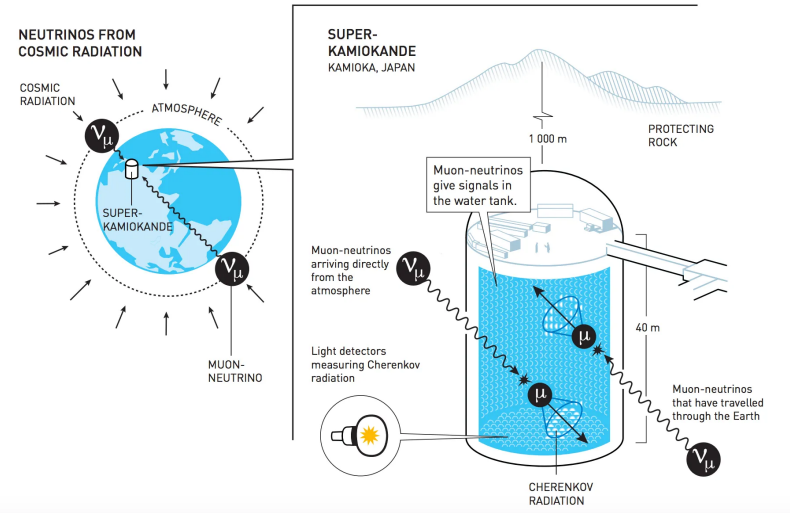


Figure 10: An infographic showing how the Super-Kamiokande neutrino experiment works. Source: nobelprize.org

- Sudbury Neutrino Observatory (SNO) (2001): Detected solar neutrinos, confirming oscillations of electron neutrinos from the Sun [29]

- KamLAND (2002): Studied electron antineutrinos from reactors, confirming oscillations and neutrino mass [30].

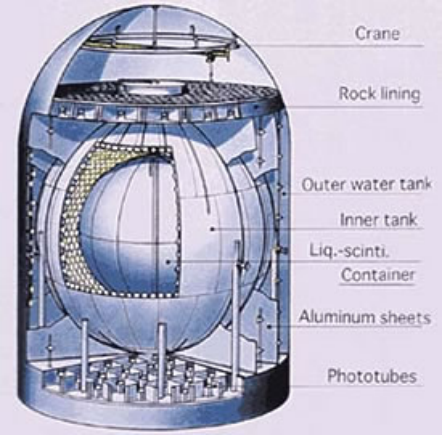


Figure 11: KamLAND (from [56])

- MINOS (2005): Measured muon neutrinos from Fermilab, providing evidence for muon neutrino oscillations [31].
- T2K (2011): Detected electron neutrino appearance from muon neutrino beams, confirming oscillations over long distances [32].
- NOvA (2015): Observed oscillations of muon neutrinos into electron neutrinos over a distance [33].

These experiments collectively demonstrate that neutrinos have mass and undergo flavor oscillations, challenging and enriching our understanding of particle physics and astrophysics.

This diagram shows cosmic rays interacting with an air nucleus in the atmosphere, producing atmospheric neutrinos[34]

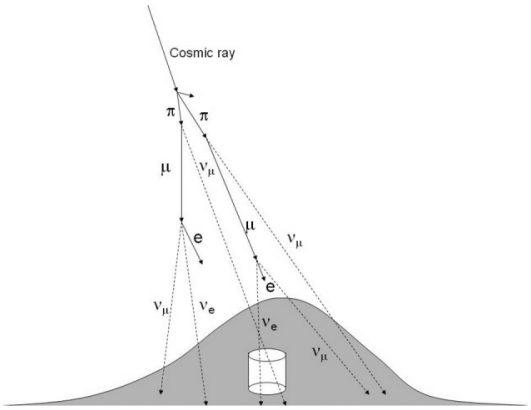


Figure 12: The cosmic rays

In fact, the first strong evidence for neutrino oscillations was obtained at Kamiokande [35] in the early **1990s** using atmospheric neutrinos. Atmospheric neutrinos come mainly from the decay of pions and muons produced when cosmic rays (high-energy protons from outer space) hit air molecules in the upper atmosphere:

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \upsilon_\mu \ , \qquad \mu^+ \rightarrow e^+ + \upsilon_e + \bar{\upsilon}_\mu, \\ \pi^- &\rightarrow \mu^- + \bar{\upsilon}_\mu \ , \qquad \mu^- \rightarrow e^- + \bar{\upsilon}_e + \upsilon_\mu \end{aligned} \tag{3.32}$$

Evidently, there should be twice as many muon neutrinos (and antineutrinos) as electron neutrinos. In fact, however, Kamiokande found roughly equal numbers of electron and muon neutrinos. This

suggests that the muon neutrinos are converting to a different flavor. Indeed, the Kamiokande detector was able to sense the direction from which the neutrinos came; those from directly overhead, which had traveled only $10km$ or so, arrived in the expected ratio $(2:1)$, but as the zenith angle increased (and with it the distance to the source), the ratio decreased . These results were confirmed and improved by SuperK in **1998** (using atmospheric neutrinos) [36].

The ideal test of neutrino oscillations would involve a fixed source (a reactor or an accelerator) and a movable detector. As the separation increases, one would monitor the sinusoidal variation predicted by Equation

$$P\left(\nu_e \to \nu_\mu\right) = \left[\sin\left(2\theta\right)\sin\left(\frac{\left(m_2^2 - m_1^2\right)c^3}{4\hbar E}x\right)\right]^2 \tag{3.33}$$

. Unfortunately, neutrino detectors tend to be huge, and oscillation lengths are typically in the range of hundreds of kilometers (while the flux from a point source falls off like $1/r^2$). So one must make do with fixed targets and extremely intense sources, and study the variation with energy.

Neutrino oscillations in matter In contrast the propagation in vacume, when neutrinos travel through matter, they experience interactions that modify their oscillation behavior. This phenomenon is known as the **Mikheyev–Smirnov–Wolfenstein effect** (MSW effect) [58]. This effect arises because electron neutrinos can interact with electrons within the matter via charged-current interactions mediated by W bosons, in addition to neutral-current interactions mediated by Z bosons. Muon and tau neutrinos, on the other hand, only experience neutral-current interactions.

These additional interactions for electron neutrinos effectively modify the potential energy of the electron neutrino component, causing a modification of the effective mixing angle and the effective mass-squared differences as the neutrinos propagate through matter [58].

3.1.2 The Dirac Neutrino

Neutrinos in the Standard Model of the Universe are defined as Left-handed (LH) massless Wely fermions.

the discovery of neutrino flavour oscillations implies nonzero neutrino masses. A simple way of explaining non-zero neutrino masses is to incorporate massless right-handed (RH) neutrinos into the SM that are singlet under the SM gauge group. In general, a fermion mass is included in the Lagrangian through the Dirac mass term

$$m\bar{\psi}\psi \tag{3.34}$$

If we take the field ψ and decompose it into left and right chiral states ψ_L *and* ψ_R respectively , (where $\bar{\psi}_R\psi_R = 0$ *and* $\bar{\psi}_L\psi_L = 0$) and The Dirac field can be decomposed into left-handed (ψ_L) and right-handed (ψ_R) components:

$$\psi = \psi_R + \psi_L \tag{3.35}$$

we can write

$$m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L \tag{3.36}$$

As seen in equation 3.34, a non-zero mass term requires both left and right-handed chiral states.

The Dirac mass term can be written as:

$$\mathcal{L}_{Mass}^{Dirac} = -m_D\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \tag{3.37}$$

where m_D the Dirac mass , This term indicates that the particle is its own antiparticle since it intrinsically relates the field to itself and preserving lepton number.

Yukawa term that couples LH and RH neutrinos to the SM Higgs doublet

$$\mathcal{L}_N^D = \frac{i}{2}\bar{N}_I^R\partial\!\!\!/ N_I^R - y_{I\alpha}\bar{N}_I^R\tilde{\phi}^\dagger L_\alpha + h.c., \qquad I = 1,2,3, \qquad \alpha = e,\mu,\tau. \tag{3.38}$$

Here, $y_{I\alpha}$ is a matrix of complex Yukawa couplings, $L_\alpha = \left(v_\alpha^L, l_\alpha^L\right)^T$ represents the SM LH lepton doublet of flavor α, and $\tilde{\phi} = i\sigma_2\phi^* = \left(\phi_0^*, -\phi_-\right)^T$ denotes the hypercharge-conjugated Higgs doublet.3.38 sets the stage for neutrino mass generation via the standard Higgs mechanism. Upon electroweak symmetry breaking (EWSB), the Higgs field acquires a nonzero vacuum expectation value (VEV), $\sqrt{2}\langle\phi_0\rangle = v \simeq 246GeV$ [59], such that LH and RH neutrinos combine into massive Dirac fermions.

3.1.3 The Majorana Neutrino

In early 1930, Ettore Majorana wondered if he could build a mass term with only a left-handed chiral state to omit the "sterile" right-handed neutrino state requirement .

He found that if we require the right-hand chiral state to be

$$\psi_R = C\overline{\psi_L}^T \tag{3.39}$$

Then we can write the field only in terms of ψ_L

$$\psi = \psi_R + \psi_L = C\overline{\psi_L}^T + \psi_L = \psi_L + \psi_L^C \tag{3.40}$$

where we have defined $\psi_L^C = C\overline{\psi_L}^T$Equation 3.39 implies that the field and its charge conjugate are the same. Thus, a Majorana particle is its own antiparticle.

The Majorana mass term in four-component notation is:

$$\mathcal{L}_{Mass}^{Majorana} = -\frac{1}{2}m_M\left(\overline{\psi}^c\psi + \bar{\psi}\psi^c\right) \tag{3.41}$$

where m_M the Majorana mass.

This term couples left-handed and right-handed fields, violation lepton number.

a Majorana mass term for the RH neutrinos, which results in the Lagrangian of the type-I seesaw model

$$\mathcal{L}_N^M = \frac{i}{2}\bar{N}_I^R\partial\!\!\!/ N_I^R - y_{I\alpha}\bar{N}_I^R\tilde{\phi}^\dagger L_\alpha - \frac{1}{2}\bar{N}_I^R M_{IJ}\left(N_I^R\right)^C + h.c. \tag{3.42}$$

For neutrinos to acquire mass under Higgs mechanism we would need a Yukawa interaction term involving right-handed neutrino fields

$$\begin{aligned} L_{Y,v} &= -y_v\bar{l}_R\tilde{\phi}v_R + h.c. = -y_v\bar{l}_R\frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}v_R + h.c. \\ &= -y_v\frac{v}{\sqrt{2}}\left(\bar{v}_L v_R + \bar{v}_R v_L\right) \end{aligned} \tag{3.43}$$

Neutrino Mass term:

$$L_{Y,v} = -y_v\left(\bar{L}_L\begin{pmatrix} v \\ 0 \end{pmatrix}v_R\right) = -y_v\left(\begin{pmatrix} v_l \\ l \end{pmatrix}_L\begin{pmatrix} v \\ 0 \end{pmatrix}v_R\right) \tag{3.44}$$

Left-Handed Neutrino Doublet $L_L = \begin{pmatrix} v_l \\ l \end{pmatrix}_L$ and Right-Handed Neutrino Singlet v_R . Which gives the mass term:

$$m_v = y_v\frac{v}{\sqrt{2}} \tag{3.45}$$

but , in The Standard Model does not include right-handed neutrino fields v_RThis means there is no v_R to form a Yukawa interaction term for neutrinos. $-y_v\frac{v}{\sqrt{2}}\left(\bar{v}_L v_R + \bar{v}_R v_L\right)$ cannot be generated , so Absence of Yukawa Term for Neutrinos within the Standard Model does not allow neutrinos to acquire mass because it lacks right-handed neutrino fields which are essential for forming Yukawa

interaction terms analogous to those for charged leptons and quarks. Extensions to the Standard Model are necessary to provide neutrinos with mass.

where the mass m_l of the charged lepton is:

$$m_l = y_l \frac{v}{\sqrt{2}} \tag{3.46}$$

Lagrangian for the first-generation leptons:

$$\mathcal{L}_{int} = g_e(\bar{L}\phi\bar{e}_R + \phi^\dagger \bar{e}_R L) \tag{3.47}$$

Since $L = \begin{pmatrix} v_e \\ e^- \end{pmatrix}$ and $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \bar{L}\phi = \bar{v}_{eL}\phi^+ e_L^- \phi^0$ is a $SU(2)$ invariant. ge arbitrary constant $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$,and $\tilde{\phi} = i\sigma_2\phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

3.2 The seesaw mechanism

The seesaw mechanism is a theoretical framework in particle physics that provides an explanation for the smallness of neutrino masses compared to other elementary particles. It postulates the existence of heavy right-handed neutrinos, known as sterile neutrinos, which interact with the standard, left-handed neutrinos through Yukawa interactions.

The modest mass of neutrinos in relation to other basic particles can be explained by the seesaw process, a theoretical paradigm in particle physics. It makes the assumption that sterile neutrinos, which are heavy right-handed neutrinos, exist and interact with ordinary left-handed neutrinos via Yukawa interactions.

3.2.1 Type I Seesaw Mechanism

In particle physics, the tiny masses of neutrinos are explained by the Type-I seesaw process. The addition of heavy right-handed Majorana neutrinos to the Standard Model (SM) of particle physics makes it more expansive.

Neutrinos are described by massless left-handed (LH) Weyl fermions in the Standard Model (SM).The inclusion of RH neutrinos (RHNs) in the theory subsequently permits the expression of a Yukawa term coupling LH and RH neutrinos to the SM Higgs doublet $\phi = \left(\phi_+, \phi_0\right)^T$.

$$\mathcal{L}_N^D = \frac{i}{2}\bar{N}_I^R \partial\!\!\!/ N_I^R - y_{I\alpha}\bar{N}_I^R \tilde{\phi}^\dagger L_\alpha + h.c., I = 1,2,3, \alpha = e,\mu,\tau. \tag{3.48}$$

Where For simplicity, the Type I Seesaw Lagrangian only incorporates two RHN and N_I^R $(I = 1,2)$ [39], $y_{I\alpha}$ in 3.49 is a rank-2 Yukawa matrix.

The Dirac-neutrino model deficiencies lead to the Lagrangian of the type-I seesaw mode in 3.48 when it is extended with a Majorana mass term for the RH neutrinos.

$$\mathcal{L}_N^M = \frac{i}{2}\bar{N}_I^R \partial\!\!\!/ N_I^R - y_{I\alpha}\bar{N}_I^R \tilde{\phi}^\dagger L_\alpha - \frac{1}{2}\bar{N}_I^R M_{IJ}\left(N_I^R\right)^C + h.c. \tag{3.49}$$

Let us now review the mechanism of SM neutrino mass generation in the type-I seesaw model. In the course of Electroweak Symmetry Breaking $(EWSB)$, the SM Higgs doublet develops a non-vanishing VEV, $\sqrt{2}\langle\phi_0\rangle = v \simeq 246Gev$, which generates a matrix of complex Dirac masses, $(m_D)_{I\alpha}$, for the LH and RH neutrinos in 3.49

$$\mathcal{L}_M^N \overset{EWSB}{\rightarrow} \frac{i}{2}\bar{N}_I^R \partial\!\!\!/ N_I^R - \left[(m_D)_{I\alpha} + y_{I\alpha}\phi_0\right]\bar{N}_I^R v_\alpha^L + y_{I\alpha}\phi_+ \bar{N}_I^R l_\alpha^L - \frac{1}{2}\bar{N}_I^R M_{IJ}\left(N_I^R\right)^C, \tag{3.50}$$

Here, φ_0 contains the real SM Higgs boson with a mass of $m_{h0} \simeq 125GeV$ after EWSB. The Dirac mass matrix is directly proportional to the RHN Yukawa matrix, $(m_D)_{I\alpha} = y_{I\alpha}v/\sqrt{2}$ After EWSB, the Dirac and Majorana mass terms in the neutrino Lagrangian can be organized as follows[39],

$$\mathcal{L}_M^N \supset -\frac{1}{2}\left(\overline{(v_\alpha^L)^C}\,\overline{N_I^R}\right)\begin{pmatrix} 0_{\alpha\beta} & (m_D^T)_{\alpha J} \\ (m_D)_{I\beta} & M_{IJ} \end{pmatrix}\begin{pmatrix} v_\beta^L \\ (N_J^R)^C \end{pmatrix} + h.c. \tag{3.51}$$

one can identify the total neutrino mass matrix, M, to conform to a 5×5 complex symmetric matrix. By Autonne-Takagi factorization, one can diagonalize M through a unitary matrix [39],

$$\mathcal{M}_{(\alpha,I)(\beta,J)} = \begin{pmatrix} 0_{\alpha\beta} & (m_D^T)_{\alpha J} \\ (m_D)_{I\beta} & M_{IJ} \end{pmatrix} \rightarrow D_{(i,I)(j,J)}\begin{pmatrix} D_{ij}^v & 0_{ij} \\ 0_{IJ} & D_{IJ}^N \end{pmatrix}, \tag{3.52}$$

where D^v and D^N contain three light and two heavy Majorana mass eigenvalues, respectively i.e. [39]

$$D_{ij}^v = m_i\delta_{ij}, D_{IJ}^N = M_I'\delta_{IJ}. \tag{3.53}$$

In Electroweak Symmetry Breaking ($EWSB$), the SM Higgs doublet archives a nonzero VEV, $\sqrt{2}\langle\phi_0\rangle = v \simeq 246Gev$, which produces a matrix of complex Dirac masses,$(m_D)_{I\alpha}$, for LH and RH neutrinos.[39] , the Lagrangian with a Type I Seesaw with m ($m \succeq 2$) singlet RH neutrinos N_{Ri} [40] ,

$$\mathcal{L} \supset \frac{i}{2}\bar{N}_{Ri}\partial N_{Ri} - \frac{1}{2}M_i\overline{N_{Ri}^C}N_{Ri} - \epsilon_{ab}Y_{\alpha i}\overline{N_{Ri}}l_\alpha^a H^b - h.c. \tag{3.54}$$

where M_i are the Majorana masses of the RH neutrino,$l_\alpha = (v_{\alpha L}, v_\alpha^-)$ with $\alpha = e, \mu, \tau$ are the LH lepton doublet and $H = (H^+, H^0)$ is the Higgs doublet. Further,$\epsilon_{ab} = -\epsilon_{ba}$ with $\epsilon_{12} = 1$. This difference is induced if CP is violated through $1 - loop$ corrections by a Higgs particle.

3.2.2 Type II Seesaw Mechanism:

To explain the smallness of the known neutrino masses, the Type II Seesaw Mechanism offers a natural and well-motivated paradigm. In this case, an $SU(2)_L$ triplet scalar Δ with a hypercharge of 1 is the smallest extension of the SM scalar sector. The following parameters apply to the $SU(2)_L$ scalar triplet $\Delta \sim (1, 3, 1)$ and SM doublet Higgs$H \sim (1, 2, 1/2)$ and The matrix representation of the triplet reads as:

$$H = \begin{pmatrix} h^+ \\ h \end{pmatrix},\ \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \tag{3.55}$$

where h and Δ^0 are the neutral components of H and Δ, respectively. Further, the charged components of the triplet Higgs, Δ^+ and Δ^{++}, have a significant role in collider experiments and are integral to discovering a Type II Seesaw.

Adding a triplet Higgs will lead to new interactions involving the LH lepton doublet and the SM Higgs. Particularly, there is a new Yukawa interaction between the LH Lepton doublet L_i and the triple scalar Δ [41],

$$\mathcal{L}_{Yukawa} = \mathcal{L}_{Yukawa}^{SM} - \frac{1}{2}y_{ij}\bar{L}_i^C\Delta L_j + h.c. \tag{3.56}$$

and the scalar interactions are given by

This potential is [41]

$$\begin{aligned} V(H,\Delta) &= -m_H^2 H^\dagger H + \lambda_H\left(H^\dagger H\right)^2 + m_\Delta^2 Tr\left(\Delta^\dagger\Delta\right) + \lambda_1\left(H^\dagger H\right)Tr\left(\Delta^\dagger\Delta\right) + \\ &\lambda_2\left(Tr\left(\Delta^\dagger\Delta\right)\right)^2 + \lambda_3 Tr\left(\Delta^\dagger\Delta\right)^2 + \lambda_4 H^\dagger\Delta\Delta^\dagger H + [\mu\left(H^T i\sigma^2\Delta^\dagger H\right) \qquad (3.57) \\ &\frac{\lambda_5}{M_P}\left(H^T i\sigma^2\Delta^\dagger H\right)\left(H^\dagger H\right) + \frac{\lambda_5'}{M_P}\left(H^T i\sigma^2\Delta^\dagger H\right)\left(\Delta^\dagger\Delta\right) + h.c] + ... \qquad (3.58) \end{aligned}$$

The potential can be simplified as follows by concentrating on the neutral components of h and l with non-trivial VEVs: [41],

$$\begin{aligned} V\left(h,\Delta^0\right) &= -m_H^2\left|h\right|^2+\lambda_H\left|h\right|^4+m_\Delta^2\left|\Delta^0\right|^2+\lambda_\Delta\left|\Delta^0\right|^4+\lambda_{H\Delta}\left|h\right|^2\left|\Delta^0\right|^2 \\ &\quad -\left(\mu h^2\Delta^{0*}+\frac{\lambda_5}{M_p}\left|h\right|^2h^2\Delta^{0*}+\frac{\lambda_5'}{M_p}\left|\Delta^0\right|^2h^2\Delta^{0*}+h.c.\right)+... \end{aligned} \tag{3.59}$$

Where $\lambda_\Delta=\lambda_2+\lambda_3$ and $\lambda_H=\lambda_1+\lambda_4$.

It is the essential relation in the seesaw scenario type II.

The VEV of Δ can be obtained from the potential in equation 3.59. The triplet Higgs non-vanishing VEV can be found to be [41], in the case $m_\Delta \geq v_{EM}$, where the SM Higgs VEV is significantly lower than the triplet Higgs mass parameter.

$$v_\Delta=\left\langle\Delta^0\right\rangle\simeq\frac{\mu v_{EW}^2}{2m_\Delta^2}, \tag{3.60}$$

where $v_{EM}=246GeV$ is the SM Higgs VEV. Moreover, investigations on the doubly-charged Higgs in the Large Hadron Collider(LHC) have yielded a lower limit, $m_\Delta \geq 800GeV$ [42].The masses of the charged and neutral components of Δ are roughly equal to $m_{\Delta^+}\simeq m_{\Delta^0}\simeq m_\Delta$,For every $m_\Delta \succ 800GeV$.

The neutrino mass matrix derived from the Yukawa interaction in equation 3.59 is as follows [41].

$$m_{ij}^v=y_{ij}v_\Delta \tag{3.61}$$

The smallness of neutrino masses is explained by a theoretical framework in particle physics called the Type II Seesaw Mechanism. The Standard Model of particle physics, which explains the interactions between elementary particles, is expanded upon by this theory.

The seesaw term involves the interaction of the leptonic doublets (L) with the Higgs triplet (Δ) or quadruplet (Φ). It can be written as:

$$\mathcal{L}_{mass}=-\frac{1}{2}\left(\overline{v_L}\overline{v_L^c}\right)\begin{pmatrix}0 & m_D\\ m_D^T & M\end{pmatrix}\begin{pmatrix}v_L\\ v_R^c\end{pmatrix}+h.c. \tag{3.62}$$

3.2.3 Type-III Seesaw Mechanism

To account for the smallness of neutrino masses, the Standard Model is further extended by the Type-III seesaw process. Triplet fermions are a new kind of fermionic fields added into this mechanism. The term triplet refers to these fermions since they are in the adjoint representation of the gauge group SU(2).

The standard model (SM) is extended in the Type-III seesaw mechanism by include fermionic triplets with zero hypercharge. These recently discovered particles are frequently designated as Σ. The crucial characteristic of the Type-III seesaw mechanism is that these fermionic triplets transform as triplets under the SU(2) gauge group of the SM instead of being singlets as in the Type-I seesaw.

To create the Seesaw neutrino mass matrix in the Type III Seesaw Mechanism, a triplet lepton representing $\Sigma:(1,3,0)$is introduced [43]. In addition to the SM particles, this model includes three LH triplets of leptons, each of which has a hypercharge of 0.

$$\Sigma=\begin{pmatrix}N^0/\sqrt{2} & E^+\\ E^- & -N^0/\sqrt{2}\end{pmatrix},\Sigma^C=\begin{pmatrix}N^{0C}/\sqrt{2} & E^{-C}\\ E^{+C} & -N^{0C}/\sqrt{2}\end{pmatrix} \tag{3.63}$$

The renormalizable Lagrangian related to Σ is [43]

$$\mathcal{L}=Tr\left[\bar{\Sigma}iD\Sigma\right]-\frac{1}{2}Tr\left[\bar{\Sigma}M_\Sigma\Sigma^C+\overline{\Sigma^C}M_\Sigma^*\Sigma\right]-\tilde{H}^\dagger\bar{\Sigma}\sqrt{2}Y_\Sigma L_L-\overline{L_L}\sqrt{2}Y_\Sigma^\dagger\Sigma\tilde{H}+h.c. \tag{3.64}$$

Defining $E = E_R^{+C} + E_R^-$,one obtains the Lagrangian,

$$\begin{aligned}\mathcal{L} &= \bar{E}i\partial\!\!\!/ E + \overline{N_R^0}i\partial\!\!\!/ N_R^0 - \bar{E}M_\Sigma E - \left(\overline{N_R^0}\frac{M_\Sigma}{2}N_R^{0C} + h.c.\right) \\ &+g\left(W_\mu^+\overline{N_R^0}\gamma_\mu P_R E + W_\mu^+\overline{N_R^{0C}}\gamma_\mu P_L E + h.c.\right) - gW_\mu^3\bar{E}\gamma_\mu E \\ &-\left(\phi^0\overline{N_R^0}Y_\Sigma v_L + \sqrt{2}\phi^0\bar{E}Y_\Sigma l_L + \phi^+\overline{N_R^0}Y_\Sigma l_L - \sqrt{2}\phi^+\overline{v_L^C}Y_\Sigma^T E + h.c\right). \end{aligned} \tag{3.65}$$

Then, one can identify the neutrino mass terms in equation 3.65 to obtain [43],

$$\mathcal{L} \supset -\left(\overline{v_L^C}\,\overline{N^0}\right)\begin{pmatrix} 0 & Y_\Sigma^T v/2\sqrt{2} \\ Y_\Sigma v/2\sqrt{2} & M_\Sigma/2 \end{pmatrix}\begin{pmatrix} v_L \\ N^{0C}\end{pmatrix} + h.c, \tag{3.66}$$

an ordinary seesaw matrix. After the charged leptons in the SM mix with their charged companions in the triplet Σ, the mass matrix that is produced is as follows:

$$\mathcal{L} \supset -\left(\overline{l_R}\,\overline{E_R}\right)\begin{pmatrix} m_l & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix}\begin{pmatrix} l_R \\ E_R\end{pmatrix} + h.c. \tag{3.67}$$

Following the introduction of unitary matrices in [39], as in the Type I Seesaw model. The mass eigenvalues and mass matrix for charged leptons and neutrinos can be diagonalized [44].

$$diag\,(N) = U_0^\dagger\begin{pmatrix} 0 & Y_\Sigma^\dagger v/2\sqrt{2} \\ Y_\Sigma^* v/2\sqrt{2} & M_\Sigma^* \end{pmatrix}U_0^* = \begin{pmatrix} m_v^{diag} & 0 \\ 0 & M_N^{diag}\end{pmatrix} \tag{3.68}$$

$$diag\,(E) = U_L^\dagger\begin{pmatrix} m_l^\dagger & Y_\Sigma^\dagger v_0 \\ 0 & M_\Sigma^\dagger \end{pmatrix}U_R = \begin{pmatrix} m_l^{diag} & 0 \\ 0 & M_\Sigma^{diag}\end{pmatrix}. \tag{3.69}$$

Hence, for $j = 1, 2, 3$, the light neutrino mass eigenstates are represented by v_j. In this case, the charged leptons are donated as $E_k^\pm$ and the heavy neutral leptons as N_j.

$$m_v \approx \frac{Y_\Sigma^2 v_0^2}{2M_\Sigma}, M_N \approx M_\Sigma \tag{3.70}$$

and for charged leptons are,

$$m_l - m_l\frac{Y_\Sigma^2 v_0^2}{2M_\Sigma} \approx m_l, M_E \approx M_\Sigma \tag{3.71}$$

The mass of the heavy leptons M and $E^\pm$ is degenerate at the tree level. However, radiative corrections split this degeneracy by [44], for $M_\Sigma \geqslant 100GeV$ after EWSB.

Conclusion

The study of neutrino oscillations and the seesaw mechanism has significantly advanced our comprehension of neutrino physics, with profound implications for high energy physics (particle physics and cosmology).

Neutrino oscillations, where neutrinos oscillate between different flavor states, confirm that neutrinos is a massive particle. This phenomenon has led to precise measurements of the mixing angles and mass-squared differences, described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. The Mikheyev-Smirnov-Wolfenstein (MSW) effect explains the influence of matter on neutrino oscillations, crucial for interpreting experiments involving dense matter environments.

The seesaw mechanism also explains the smallness of neutrino masses by introducing heavy right-handed neutrino states N_R. This framework suggests new physics beyond the Standard Model and aligns well with experimental data. Variants like the Type I, Type II, and Type III seesaw mechanisms provide alternative pathways and testable predictions for future research.

The advantage of adopting the seesaw mechanism is that it preserves the general structure of the Standard Model without introducing significant changes that could disrupt its successful components. This mechanism integrates seamlessly with the existing framework, adding only minimal extensions, such as heavy right-handed neutrinos, which are singlets under the Standard Model gauge group. Consequently, it provides a natural explanation for the smallness of neutrino masses without compromising the Standard Model's predictive power and established successes. This compatibility makes the seesaw mechanism an elegant and compelling solution in the quest to understand neutrino masses and the nature of neutrinos.

Implications and Future Directions:

Understanding neutrino masses, the nature of neutrinos (Dirac or Majorana), and potential CP violation in the lepton sector are key open questions. Future experiments, such as neutrinoless double-beta decay searches and long-baseline neutrino oscillation studies, will probe the seesaw mechanism's predictions and further our understanding of neutrino physics.

In summary, the exploration of neutrino oscillations and the seesaw mechanism challenges the Standard Model and opens pathways to new physics, essential for unraveling the mysteries of neutrinos and their role in the universe.

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